Youtube:

https://youtu.be/EaHn3hV84FE

Convex Mirrors: Summary

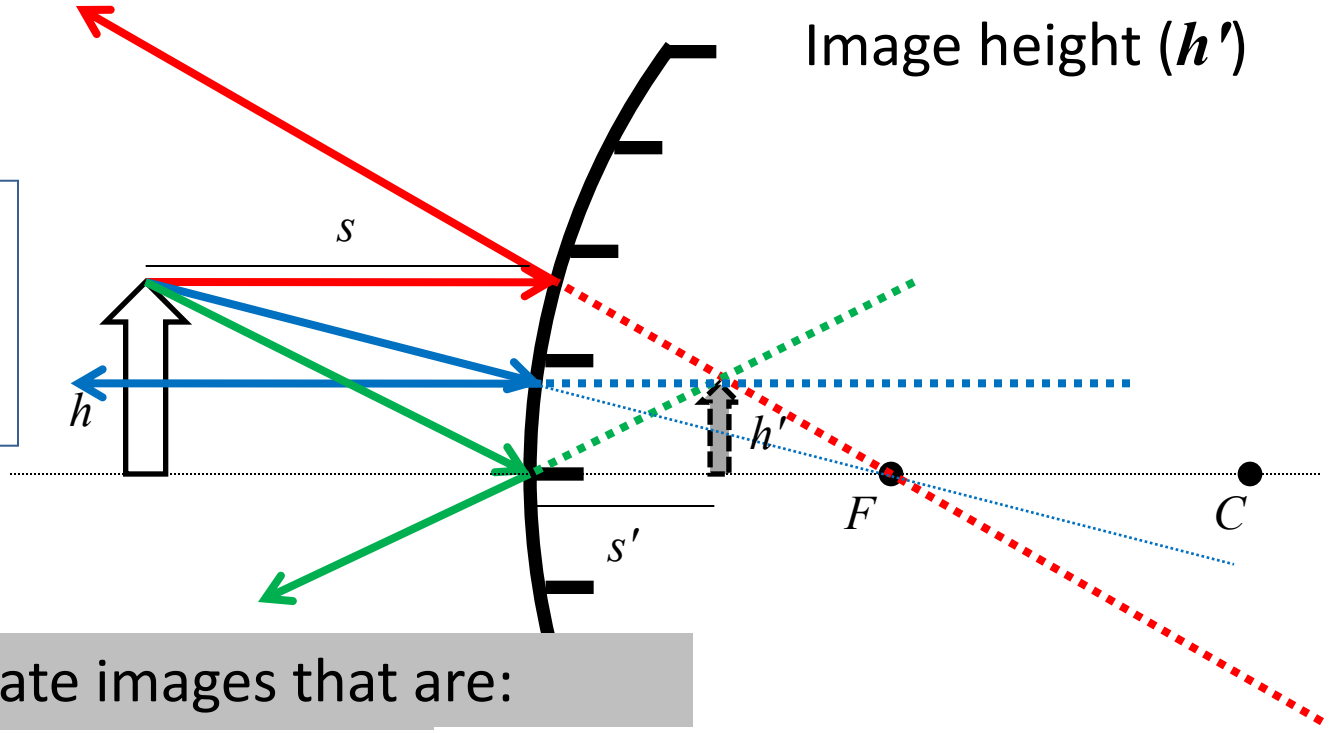
Object distance (s)

Object height (h)

Image distance (s')

Image height (h')

Focal length is called **negative**
(see slide 51: mirror equation)



Convex mirrors create images that are:

- **Upright**
  (no orientation change)
- **Reduced**
  (smaller than object)
- **Virtual**
  (formed by intersection of projected lines)

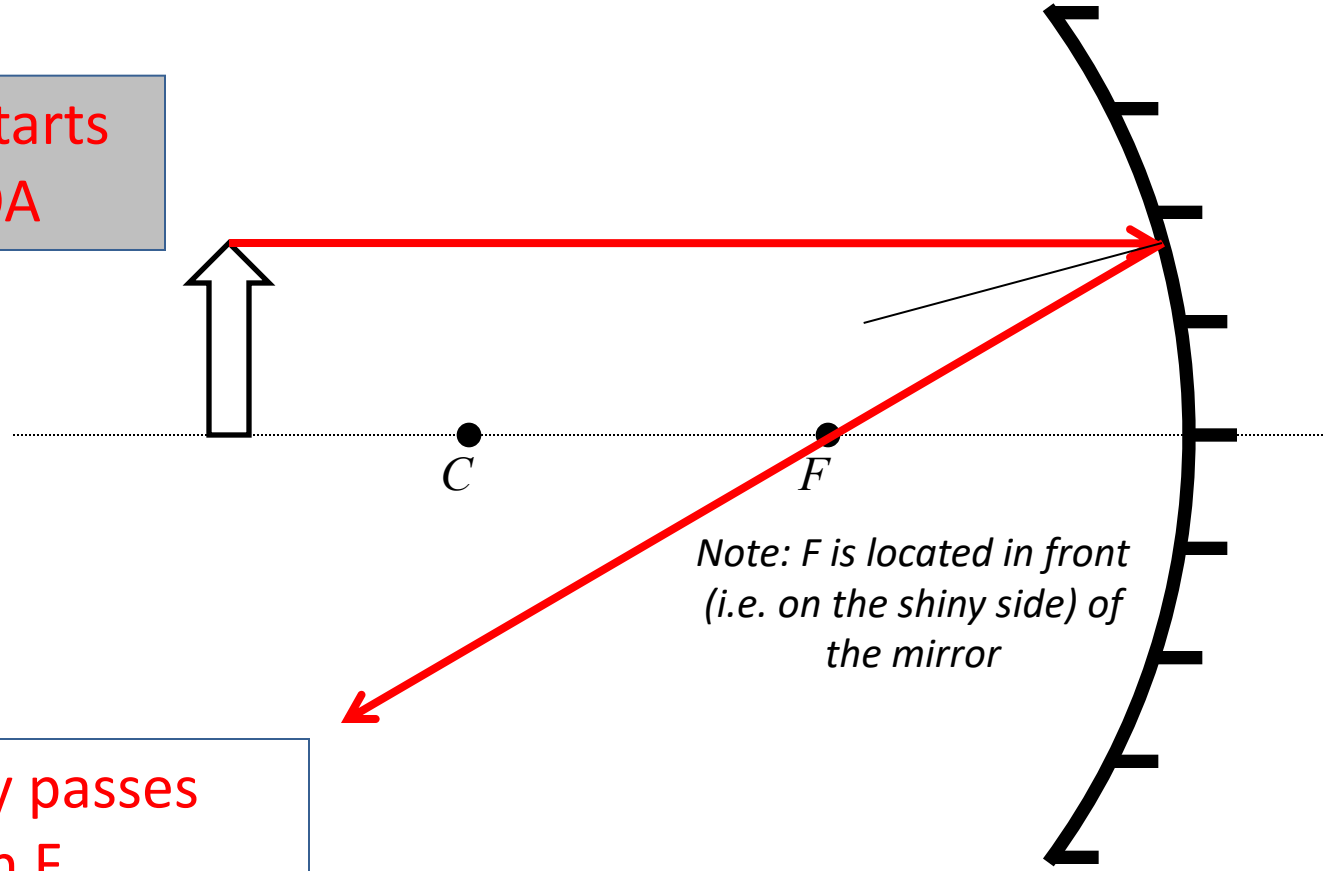
Tips for Ray Diagrams

- Make sure you use a ruler or graph paper to get distances correct if you want to use diagram to find exact distances
- Instead of trying to make sure mirror is actually spherical (circular on page)
  - Include a "**mirror plane**" (line perpendicular to OA and passing through mirror's center)
  - Draw rays reflecting off this plane
- Draw all 3 special rays to confirm your results
- Good for getting an idea of what's going on

Concave Mirrors: Ray Diagram

- Use three “Special Rays” to find images
- Each one obeys Law of Reflection

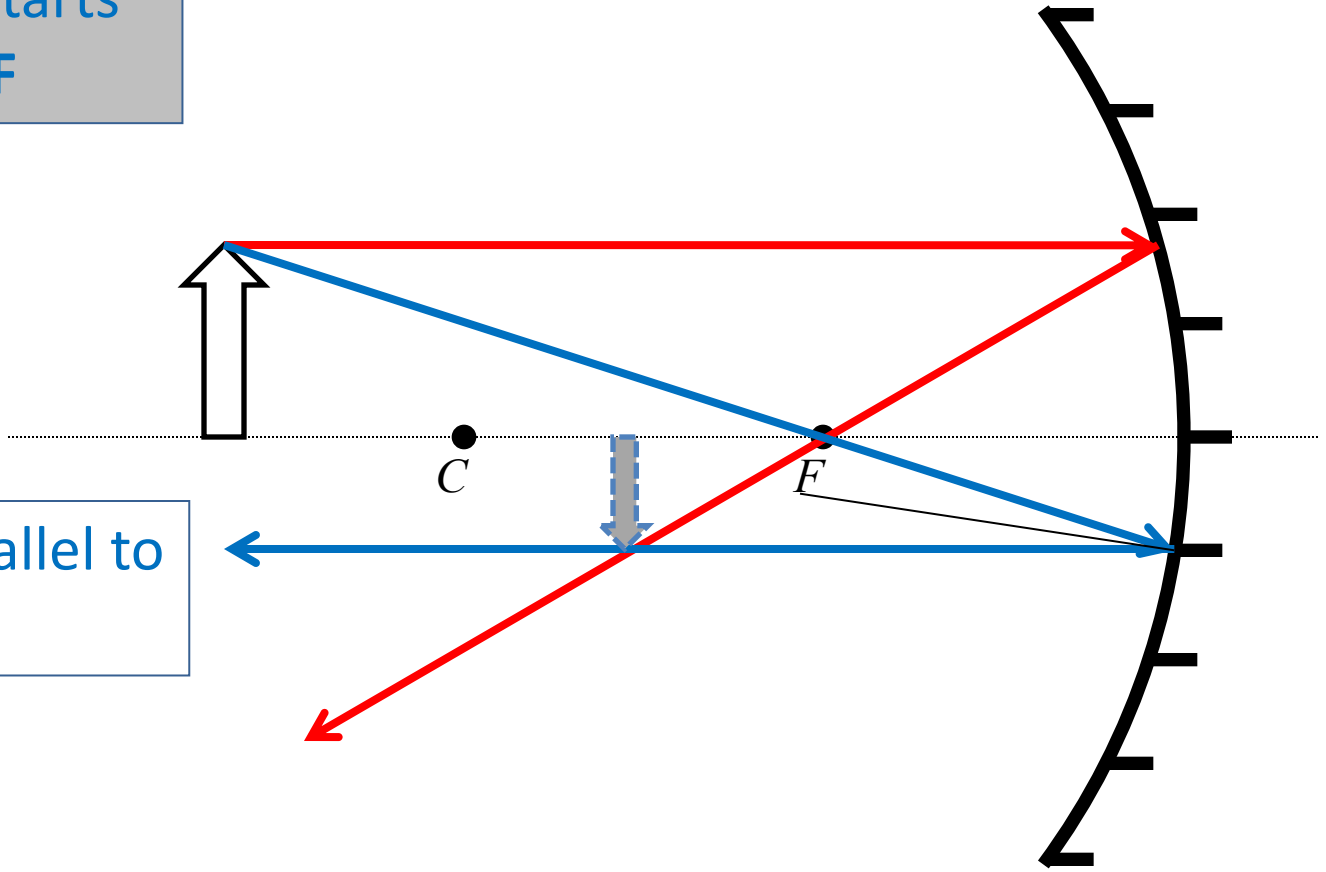
1. Ray which starts **parallel** to OA



Reflected ray passes through F

Concave Mirrors: Ray Diagram

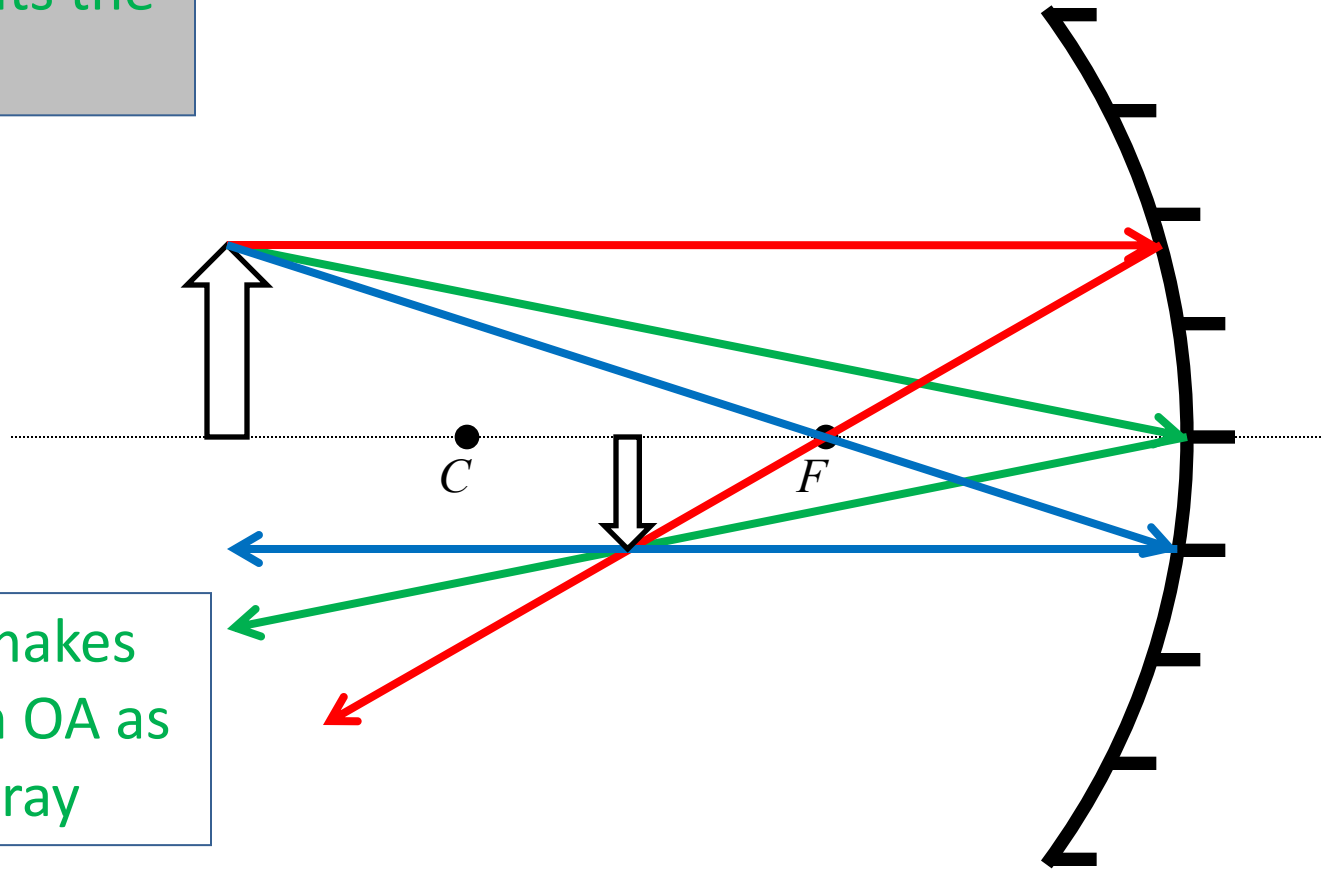
2. Ray which starts **aimed at F**



Reflected ray is parallel to the OA

Concave Mirrors: Ray Diagram

3. Ray which hits the **mirror's center**



Reflected ray makes same angle with OA as the incident ray

1 Ray which starts parallel to OA

Reflected ray passes _____

2 Ray which passes through the focal point

Reflected ray is _____

3 Ray which hits the mirror's center

Reflected ray makes _____ as the incident ray

Concave Mirrors: Summary

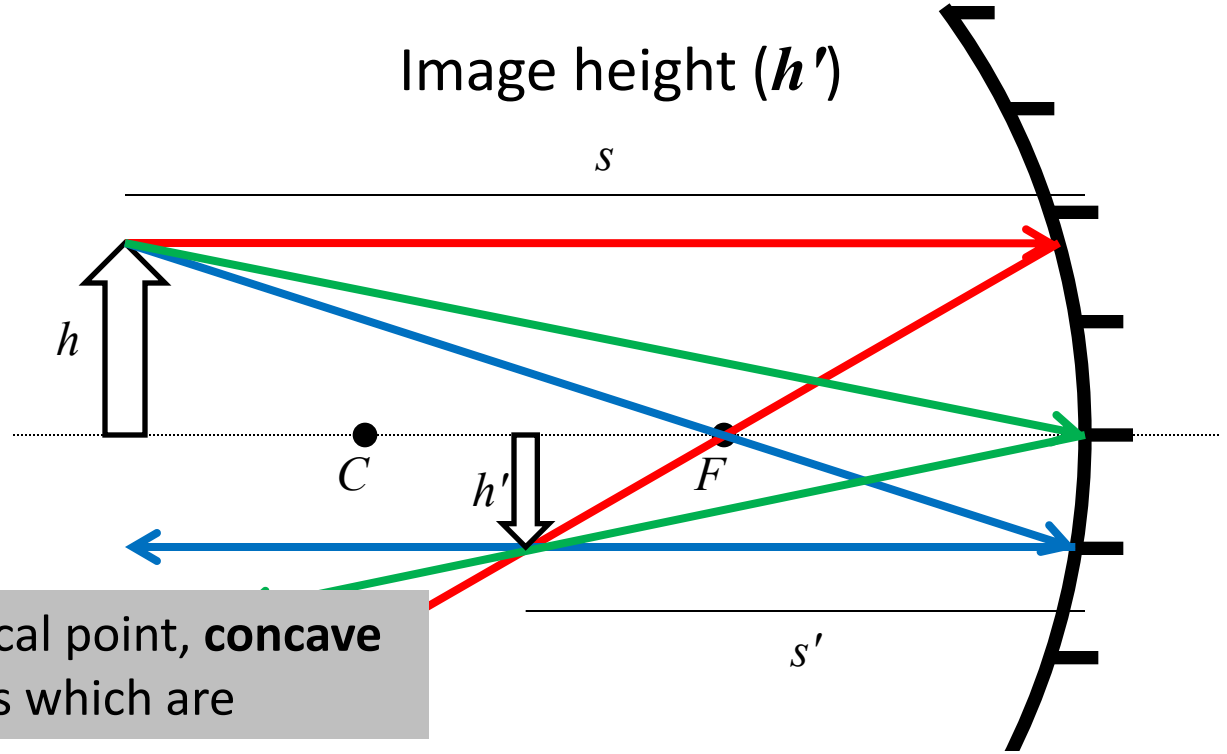
(for object located **outside F**)

Object distance (s)

Image distance (s')

Image height (h')

Object height (h)



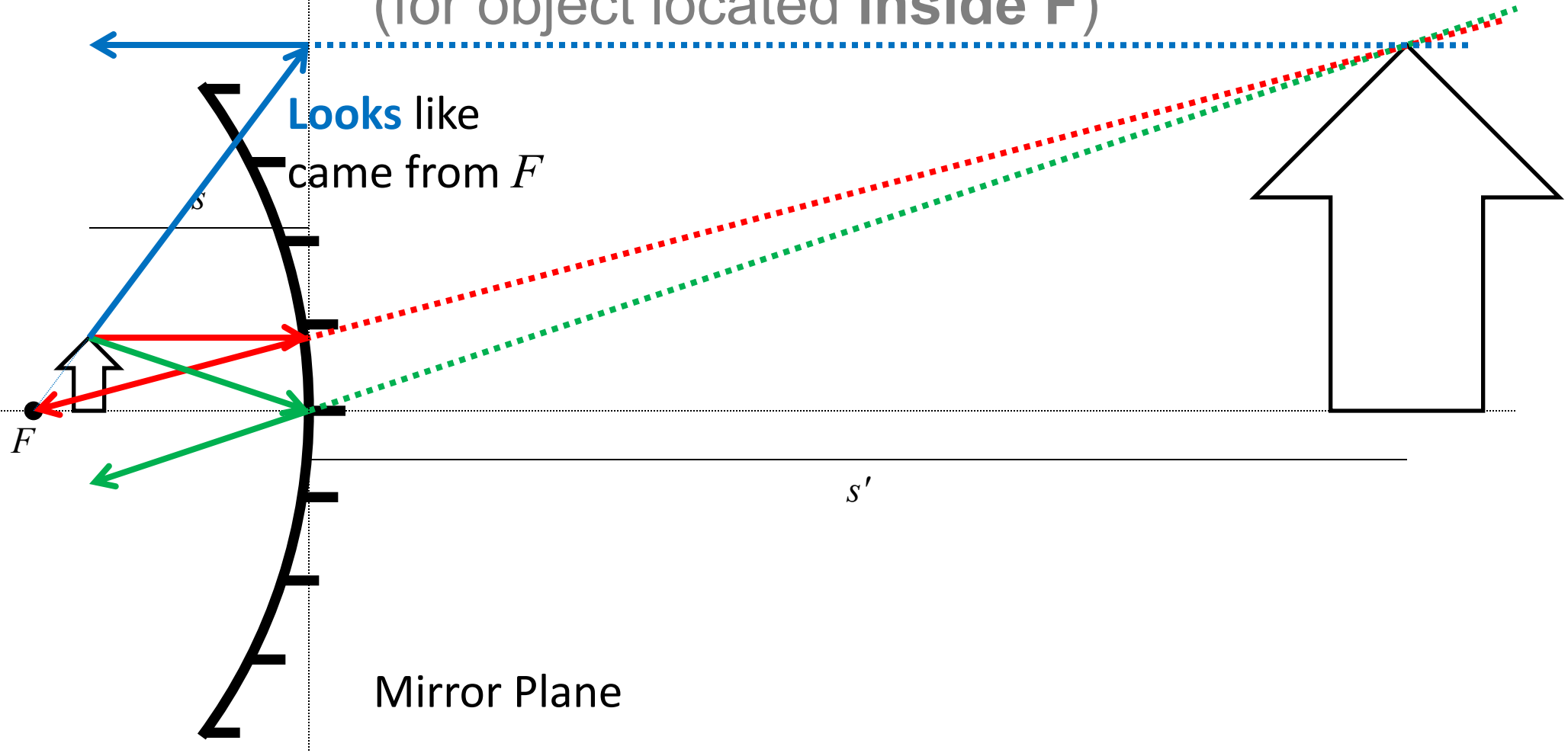
Focal length is called **positive**

For an object **outside** the focal point, **concave mirrors** create images which are

- **Inverted** (upside down)
- **Real** (formed by intersection of actual rays)
- **Can be reduced or enlarged**

Concave Mirrors

(for object located **inside F**)



- **Upright**
  (no orientation change)
- **Enlarged**
- **Virtual**
  (formed by intersection of projected rays)

Which Type of Mirror?

***A:** Convex Mirror; **B**: Concave Mirror; **C:** Plane Mirror;*
***D**: None of the above; **E:** More than one of the above*

- Want to see a upright, larger *field of view* (not the same as larger *image*)

A – Convex mirror



http://en.wikipedia.org/wiki/File:Rear-view-mirror-caption.jpg

- Want to see a magnified, upright image

B – Concave mirror

$s < f$

- Want to focus light on screen



http://web.mit.edu/2.009/www/experiments/deathray/10_ArchimedesResult.html

B – Concave mirror

$s > f$

(e.g. Flashlights Headlights)

- Want to see object as it is (not reduced or enlarged)

C – Plane mirror

Mirror Equation

- Mirror equation and Magnification
  - Use when you need exact answers
  - Image distance:
    - Positive if image is real
    - Negative if image is virtual
  - Magnification:
    - Negative if image is inverted
    - Positive if image is upright

Mirror Equation

Assuming that the mirror is fairly flat then image distance, object distance, and focal length are related by the following:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Sign Conventions:

	Positive	**Negative**
Object Distance (s)	Object is **in front** of mirror (usually)	
Image Distance (s')	Image is **real** (intersection of actual rays)	Image is **virtual** (intersection of projected rays)
Focal Length (f)	**Concave Mirror**	**Convex Mirror**

Magnification

Unlike for a plane mirror, the size of an image in a spherical mirror is different from the size of the object

Magnification is defined as:

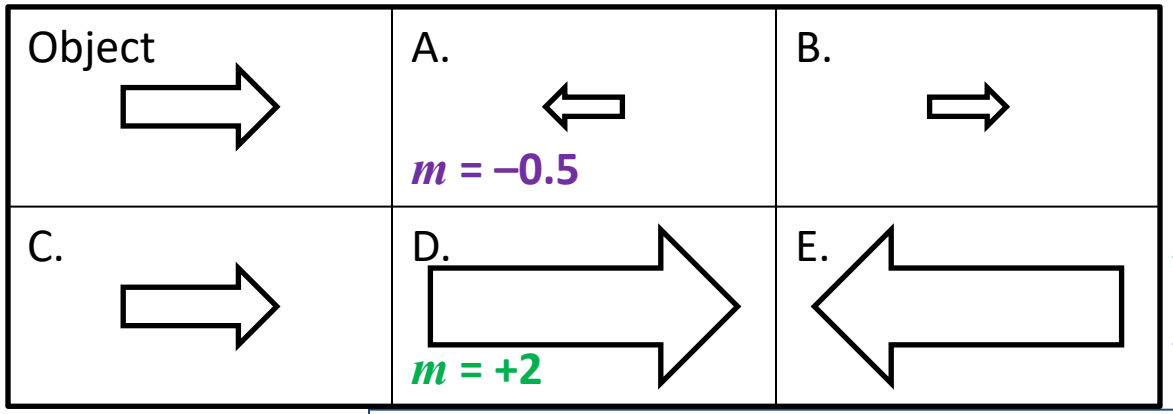
$$m = \frac{h'}{h}$$

h': Image height

h: Object height

(unitless quantity)

For this object, what's the image if $m = +2$?
If $m = -0.5$?



$|m| > 1$ means **enlarged image**

$|m| < 1$ means **reduced image**

$m > 0$ means **upright image**

$m < 0$ means **inverted image**

Using similar triangles, we can also find magnification by:

$$m = -\frac{s'}{s}$$

Convex Mirror Example

A 2.0 cm tall eraser is 3.0 cm from a convex mirror. If the eraser's image is 0.8 cm tall, what is the mirror's focal length?

A. –2 cm
~~B. +0.86 cm~~
C. –1.2 cm
D. –15 cm
~~E. +2 cm~~

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Want f, will need s' first

Given:

h = 2.0 cm

h' = +0.8 cm

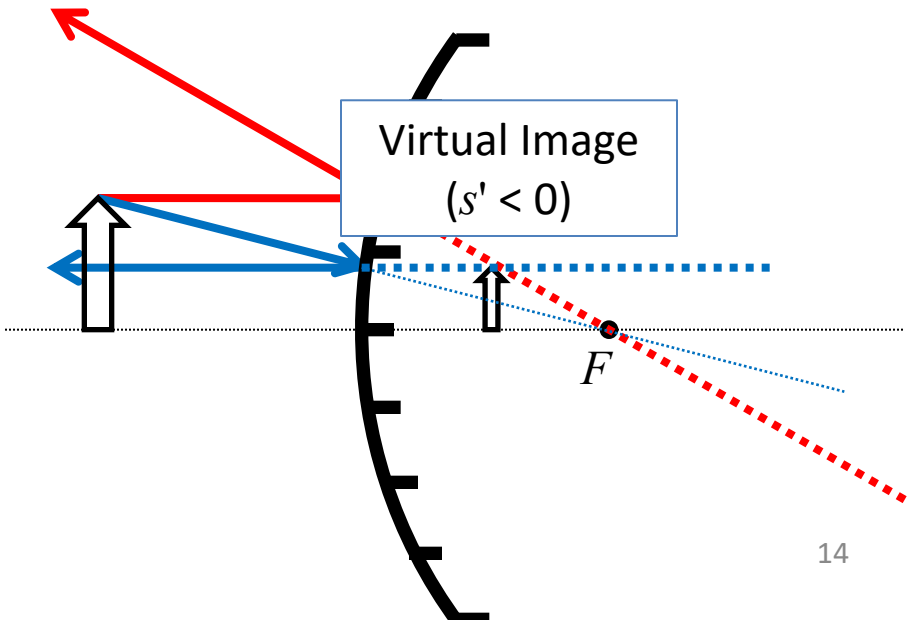
s = 3.0 cm

$$m=\frac{h'}{h}=0.4$$

$$m=-s'/s$$

What is s'?

A. –7.5 cm
B. –1.2 cm
C. –2 cm
~~D. +7.5 cm~~
~~E. +1.2 cm~~

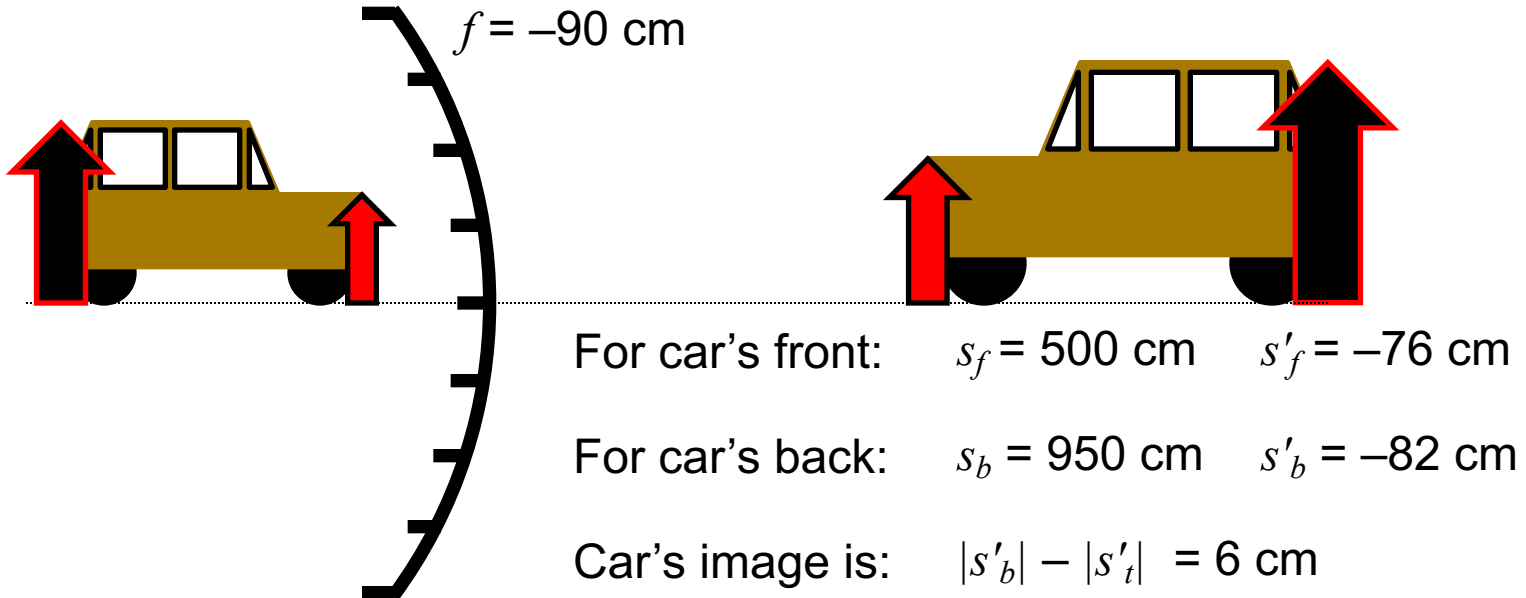


What to Do About Long Objects?

How do we deal with objects that have an extension **parallel to the optical axis?**

A car's front end is 5 m behind another car's passenger side rear view mirror. If the car is 4.5 m long and the mirror's focal length is –90 cm, how long is the car's image?

Treat **different points** as a separate objects



For car's front: s_f = 500 cm s'_f = –76 cm

For car's back: s_b = 950 cm s'_b = –82 cm

Car's image is: $|s'_b| - |s'_t|$ = 6 cm

Refraction

- OpenStax Chapter 25.3
- Refraction
  - Light changes direction when it changes medium
  - Can form image
    - (n_1 = index of medium object is in; n_2 = index of medium observer is in)
- Total Internal Reflection
  - If angle of incidence is too big, no light refracts into second medium (only happens if $n_1 > n_2$)

Changing Materials

What happens when a light ray reaches a non-opaque material?

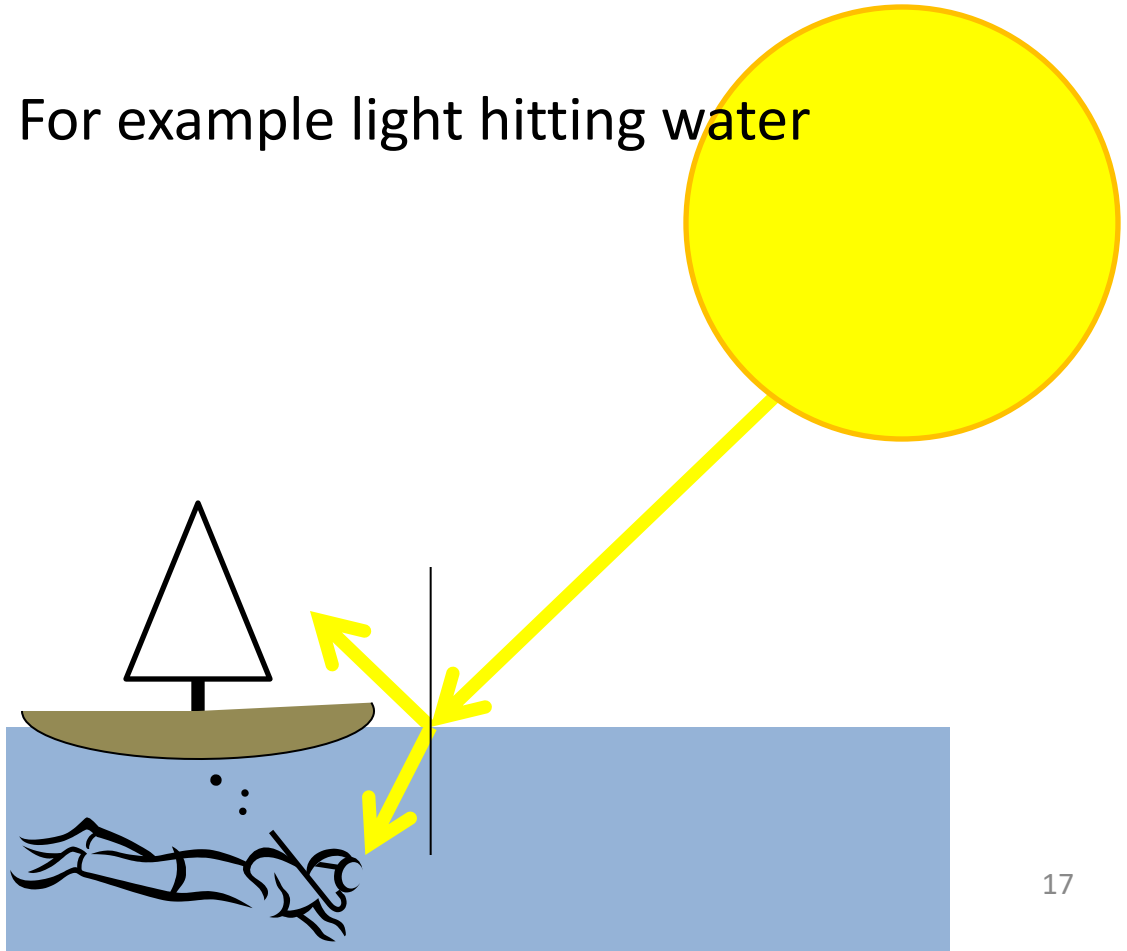
Some light reflects

Some enters the second medium

But changes direction

Process is called **Refraction**

For example light hitting water

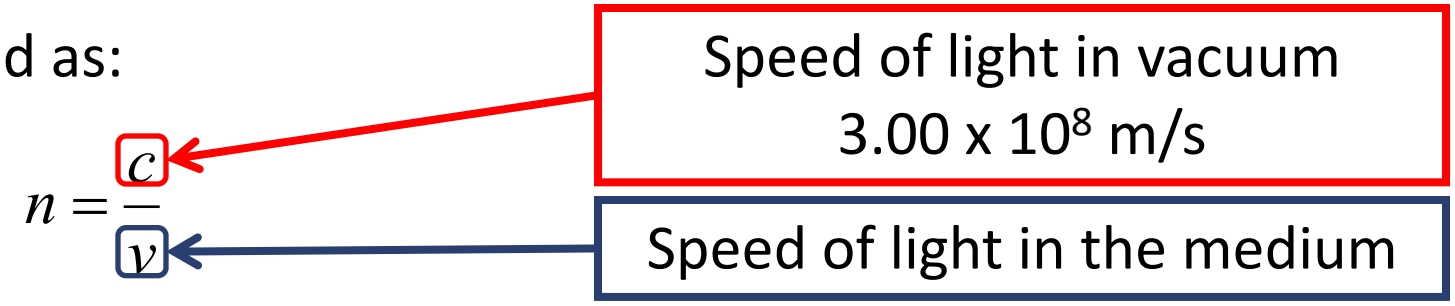


Index of Refraction

To describe refraction we will use each medium's

n = index of refraction

This is defined as:



(unitless quantity)

- Light ***never*** travels faster than it does in a vacuum

In other words:

$$c \geq v$$

$$n \geq 1$$

In air, ***n*** is close enough to 1 that we will usually use that

Indices of Refraction

From *College Physics* 2nd Edition; by Knight, Jones, and Field
© 2010 Pearson Education, Inc.

Material	Index of Refraction
Vacuum	1
Air	1.0003 ≃ 1
Water	1.33
Ethyl alcohol	1.36
Lens in human eye	1.42
Oil (typical)	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.42
Silicon (infrared)	3.50

Snell's Law

Refraction is governed by:

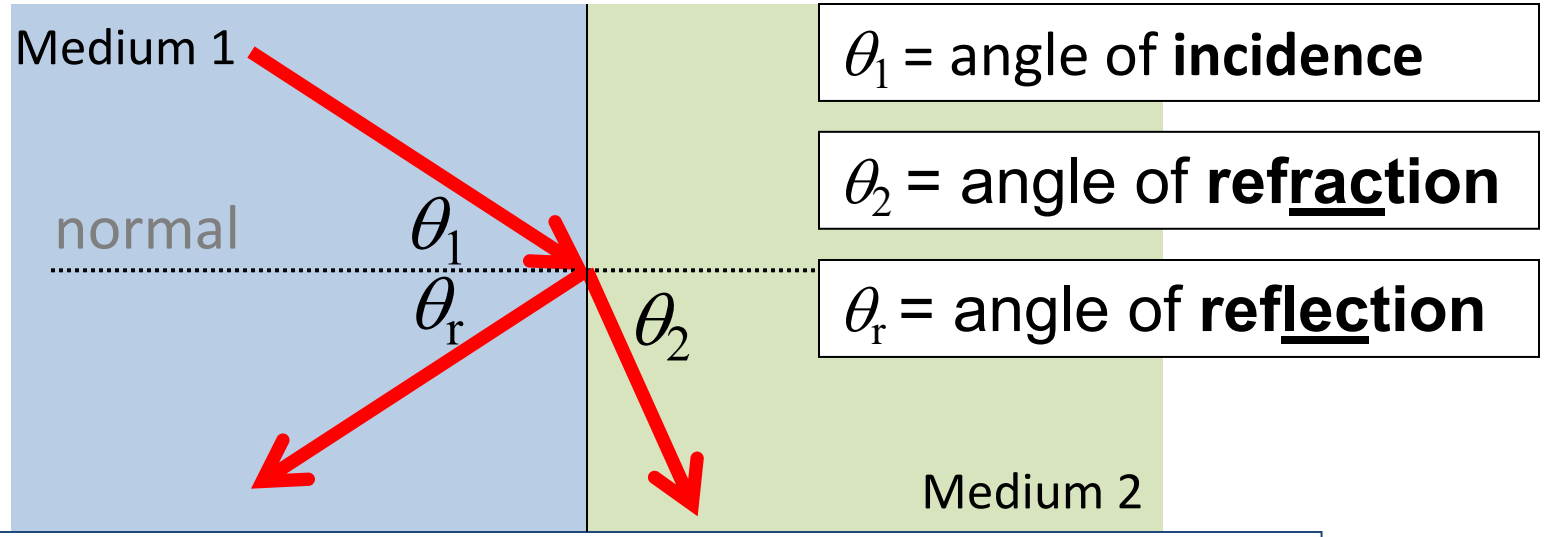
$$\underbrace{n_1 \sin(\theta_1)}_{\text{Medium 1}} = \underbrace{n_2 \sin(\theta_2)}_{\text{Medium 2}}$$

n_1, θ_1 → *medium 1*
n_2, θ_2 → *medium 2*



http://en.wikipedia.org/wiki/File:Willebrord_Snellius.jpg

*happens since light is a **wave** (see next week)*



θ_1 = angle of **incidence**

θ_2 = angle of **refraction**

θ_r = angle of **reflection**

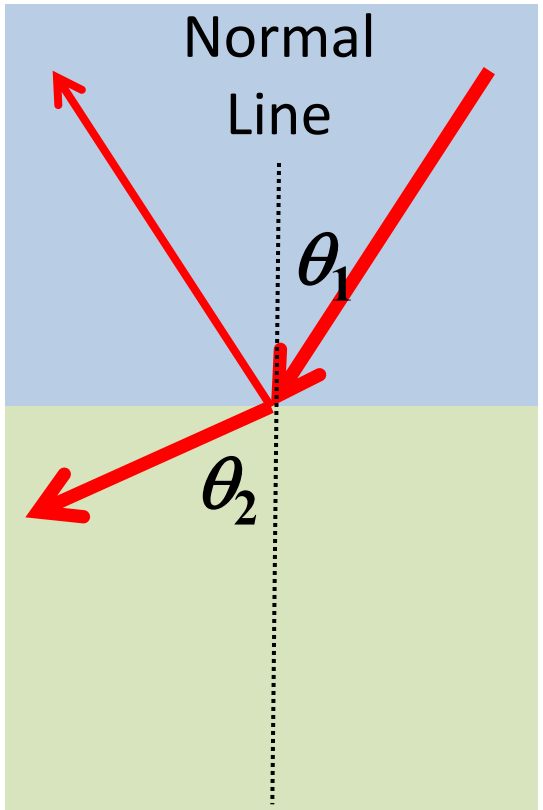
Note: Diagram assumes something about n_1 **vs.** n_2

Each Case Matched with Figure:

$n_1 > n_2$

$\theta_1 < \theta_2$

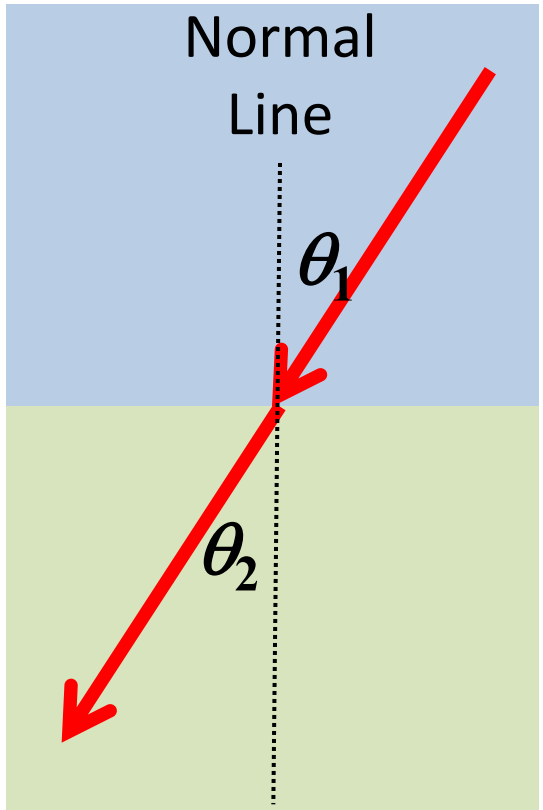
Light bends **away** from normal line:



$n_1 = n_2$

$\theta_1 = \theta_2$

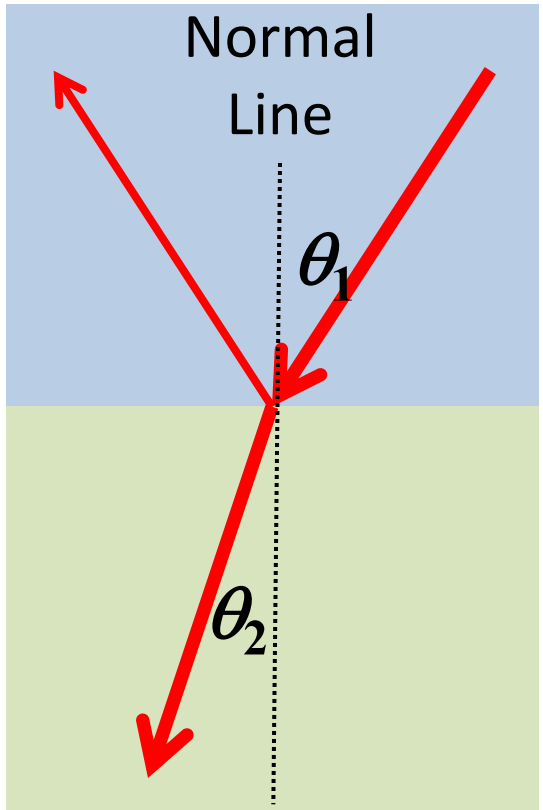
Light doesn't deflect



$n_1 < n_2$

$\theta_1 > \theta_2$

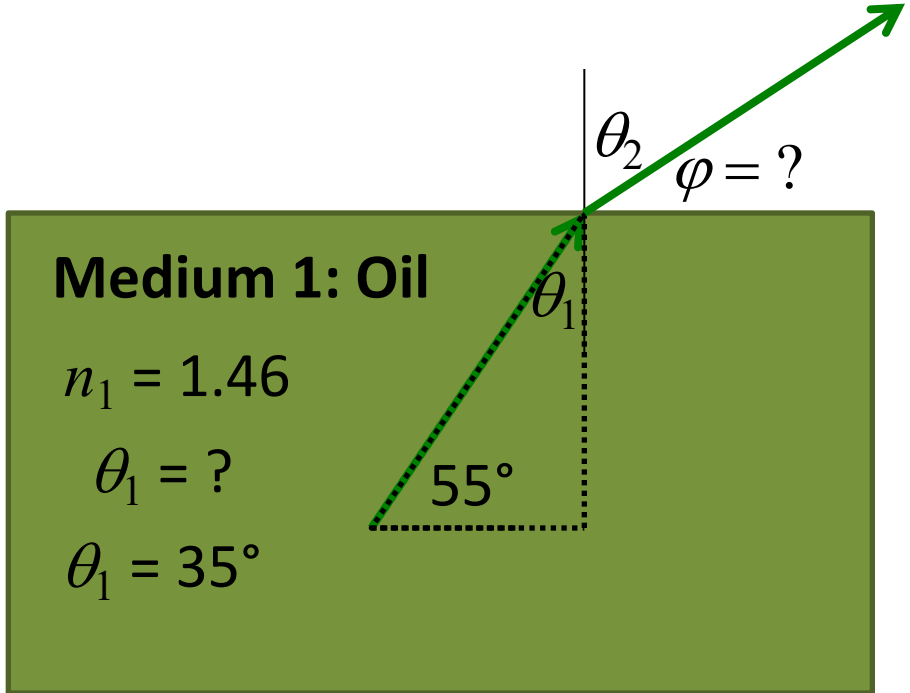
Light bends **towards** normal line:



Snell's Law and Laser Pointer

A laser pointer submerged in mineral oil (index of refraction 1.46) is aimed at 55° **above the horizontal**. What angle **above the horizontal** will the beam make after it passes through a horizontal interface with air?

A. 33° B. 45° C. 55° D. 35° E. 57°



Medium 2: Air $n_2 = 1$

θ_2 = ?

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$\Rightarrow$ $\theta_2 = \sin^{-1}(1.46 \sin(35°))$

$\Rightarrow \theta_2 \approx 57°$

$$\varphi = 90° - \theta_2 = 33°$$

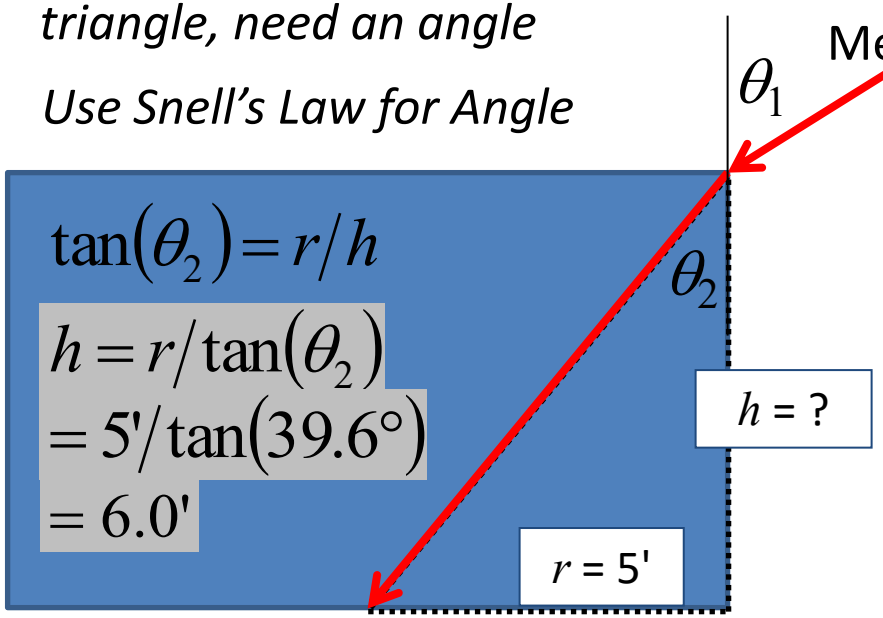
Determining Depth of a Pool

You want to find out how deep a 10.0' diameter (circular) swimming pool is by shining a laser pointer from the **rim** to the base at the **center** of the pool. If you have the laser pointer aimed (in air) at 58.0° from the vertical, how deep is the pool if it is filled to the brim with water (n = 1.33)?

- *Want one side of a right triangle, need an angle*
- *Use Snell's Law for Angle*

Medium 2: Water $n_2 = 1.33$

Medium 1: Air $n_1 = 1$ $\theta_1 = 58.0°$



$$\tan(\theta_2) = r/h$$

$$h = r/\tan(\theta_2)$$

$$= 5'/\tan(39.6°)$$

$$= 6.0'$$

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\Rightarrow \theta_2 = \sin^{-1}(n_1 \sin(\theta_1)/n_2)$$

$$= \sin^{-1}(1 \sin(58°)/1.33)$$

$$= 39.6°$$

Critical Angle and Total Internal Reflection

(OpenStax 25.4)

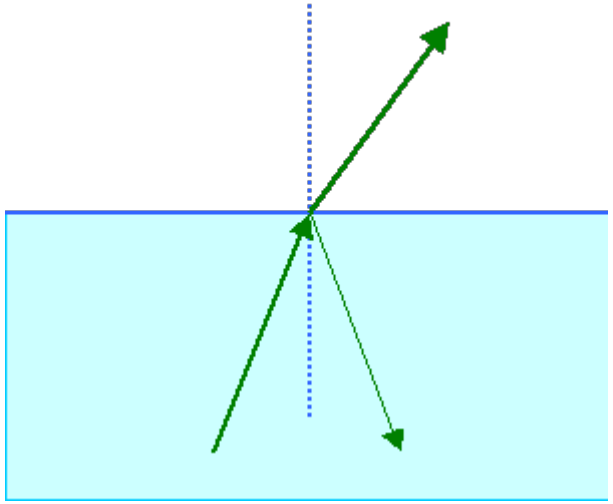
Let's look at a tank of water and shine a light in it:

- **air** to **water**, light gets refracted into the **water**
  (i.e. bends closer to the normal: $\theta_1 > \theta_2$)
- **water** to **air**, light gets refracted into the **air**
  (i.e. bends farther away from the normal: $\theta_1 < \theta_2$)

For water to air, try increasing angle of incidence:

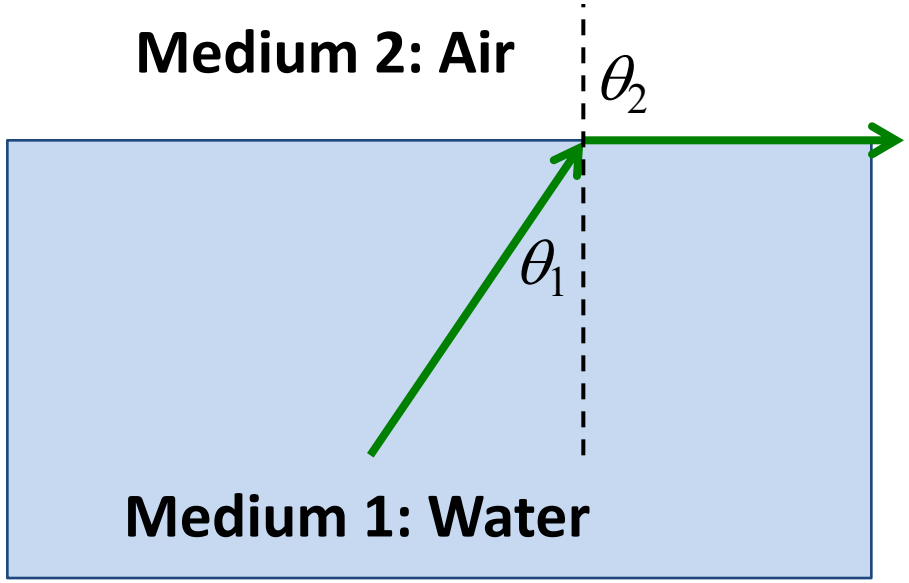
- Light bends farther and farther away from the normal, and eventually no light is refracted into the **air;**
- It refracts completely along the **surface**

This angle of incidence is called the
Critical Angle (θ_C)



Critical Angle and Total Internal Reflection

Critical angle = **incident angle** for **angle of refraction** of **90°**



$\theta_1 = \theta_C$ (critical angle)

$\theta_2 = 90°$

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\Rightarrow \theta_c = \sin^{-1}\left[\frac{n_2}{n_1}\right]$$

(*sin 90°* = 1)

(for **water** to **air** this is about 49°)

Note: only possible if n1 > n2 (because light has to bend away from the normal)

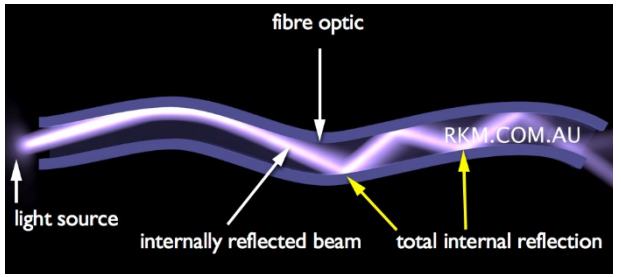
If $\theta_1 > \theta_C$: light gets completely reflected (zero refraction)

(light cannot escape from water to air; it stays in the water)

i.e. light undergoes **Total Internal Reflection**

Fiber Optics

Fiber optics cables take advantage of total internal reflection to send a light beam along a (curved) "wire" made of glass fiber

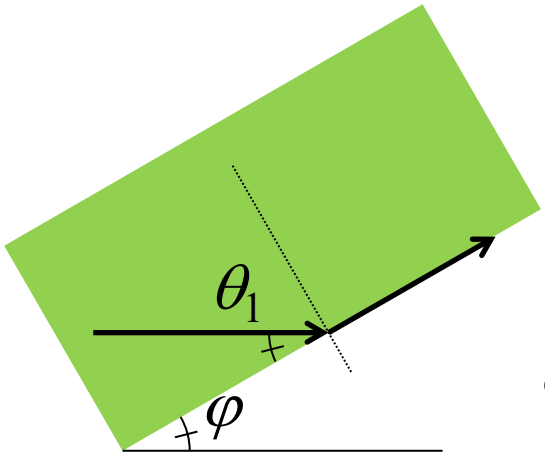


Consider two straight lengths of glass (n = 1.50) with a beam of light traveling down one. What is the biggest angle φ can be for the light to not reach the air surrounding the glass?



A. 90°
B. 27.5°
C. 41.8°
D. 48.2°
E. 62.5°

zoom in on tilted surface:



$n_2 = 1$, $n_1 = 1.5$

$$\theta_c = \sin^{-1}\left[\frac{n_2}{n_1}\right] = 41.8°$$

$$\theta_1 = \theta_c = 41.8°$$

The angle we want, φ, is the **complement** of θ_1

$$\varphi = 90° - \theta_1$$

Images formed by Refraction

- OpenStax Chapter 25.6

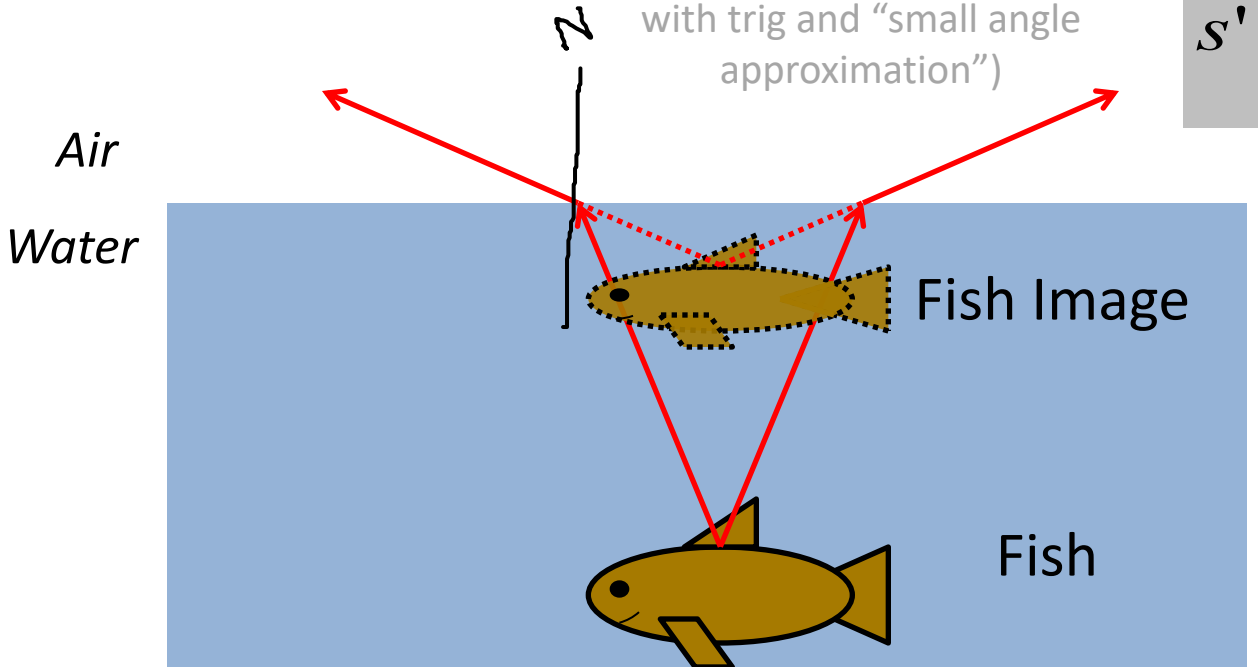
- Images formed by refracted rays
  - From flat surface
    - Image is closer to (object in higher index material) or farther from surface (object in lower index material)
  - From thin lens
    - Converging Lens: takes rays parallel to OA and puts them through focal point
    - Diverging Lens: takes rays parallel to OA and makes them look like they came from focal point
      - Can use ray diagram (3 special rays for each type of lens)
      - Thin Lens Equation

Images and Refraction

- Ray diagrams can be used to find images
- Why does depth appears different when looked at from outside of water?
- Image is formed at the point refracted rays seem to diverge from

(Image distance can be found with trig and "small angle approximation")

$$s' = \frac{n_2}{n_1} s$$



n_1 = index for medium object is in

n_2 = index for medium observer is in

Height of Rock (1)

A 3.00 cm tall rock sits at the bottom of pool of water (n = 1.33). When you look at it from above, how tall do you think it is?

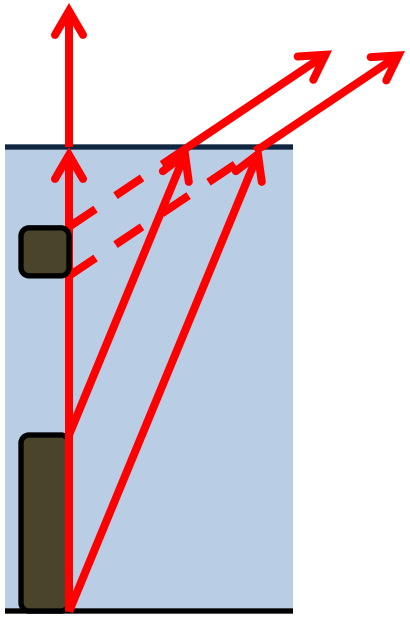
a. ~~3.00 cm~~
b. 2.26 cm
c. ~~4.51 cm~~
d. ~~3.99 cm~~
e. 1.01 cm

s_t' = the depth of the image's top

s_b' = the depth of the image's bottom

s_t = the depth of the rock's top

s_b = the depth of the rock's bottom



Height of Rock (2)

A 3.00 cm tall rock sits at the bottom of pool of water (n = 1.33). When you look at it from above, how tall do you think it is?

a. ~~3.00 cm~~
b. 2.26 cm
c. ~~4.51 cm~~
d. ~~3.99 cm~~
e. 1.01 cm

s_b = the depth of the rock's bottom

s_t = the depth of the rock's top

$h = s_b - s_t$ = 3.00 cm = rock's height

Want: h' = image's height

s_b' = the depth of the image's bottom

s_t' = the depth of the image's top

$h' = s_b' - s_t'$

Which of the following gives the depth of the bottom of the image? [see slide 14]

A. $\frac{n_{air}}{n_{H2O}} s_b$ **B.** $\frac{n_{H2O}}{n_{air}} s_b + h$ **C.** $\frac{n_{air}}{n_{H2O}} s_b + h$ **D.** $\frac{n_{air}}{n_{H2O}} s_b - h$ **E.** $\frac{n_{H2O}}{n_{air}} s_b$

Height of Rock (3)

A 3.00 cm tall rock sits at the bottom of pool of water (n = 1.33). When you look at it from above, how tall do you think it is?

Bottom of the image : $\frac{n_{air}}{n_{H2O}} s_b$

$Top\ of\ image$: $\frac{n_{air}}{n_{H_2O}} s_t$

- Which of the following gives how tall the image (h') is?

A. $\frac{n_{H2O}}{n_{air}}(s_b - s_t)$ **B.** $\frac{n_{air}}{n_{H2O}} s_b - h$ **C.** $\frac{n_{air}}{n_{H2O}} s_t - h$ **D.** $\frac{n_{air}}{n_{H2O}}(s_b - s_t)$ **E.** $\frac{n_{air}}{n_{H2O}} s_b + h$

i.e. $h' = \frac{n_{air}}{n_{H_2O}}(s_b - s_t)$,

where $s_b - s_t$ = 3 cm

$\Rightarrow\ h' = \frac{1.0}{1.33}(3)$ = **2.26 cm**

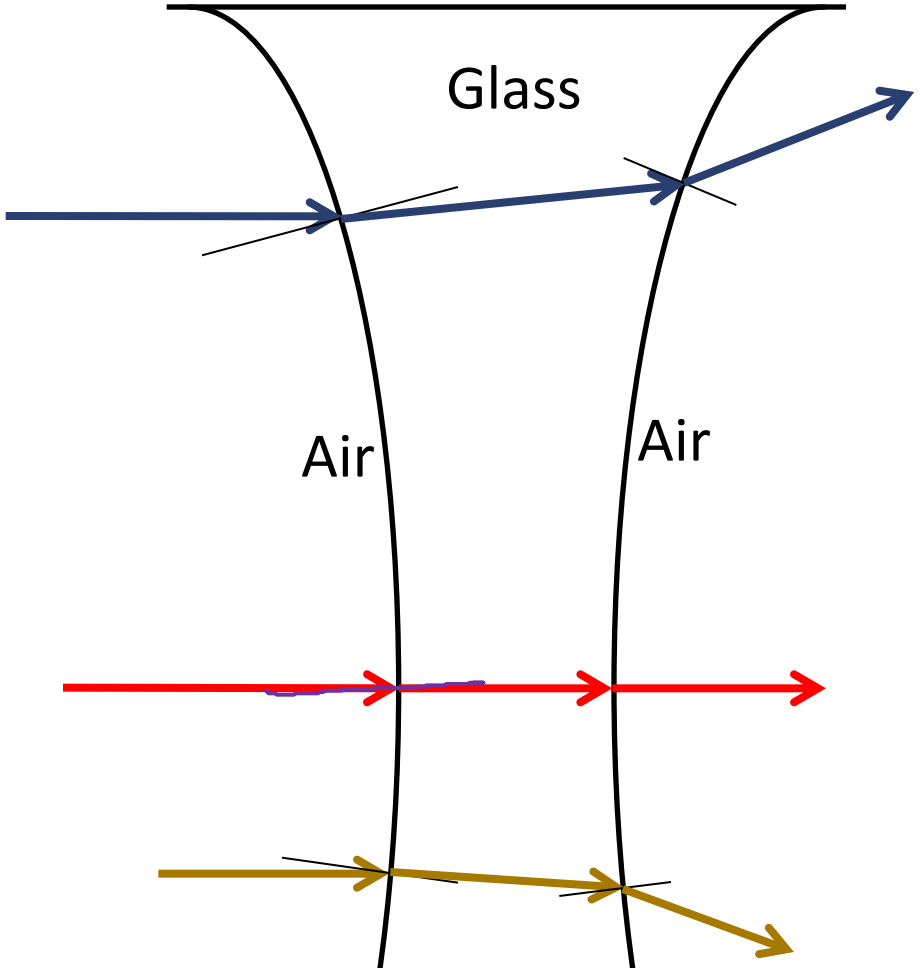
a. 3.00 cm
b. 2.26 cm
c. 4.51 cm
d. 3.99 cm
e. 1.01 cm

Lenses

- OpenStax Chapter 25.6
- Images formed by refracted rays
  - From thin lens
    - Converging Lens: takes rays parallel to OA and puts them through focal point
    - Diverging Lens: takes rays parallel to OA and makes them look like they came from focal point
      - Can use ray diagram (3 special rays for each type of lens)
      - Thin Lens Equation
- Multiple Lenses
  - Ray diagram approach: special rays look like they came from previous lens' image
  - Thin Lens approach: each image acts as next lens' object

Refraction from a Curved Surface

Just as with reflection, the normal line for refraction gets defined at the point where a ray touches the surface



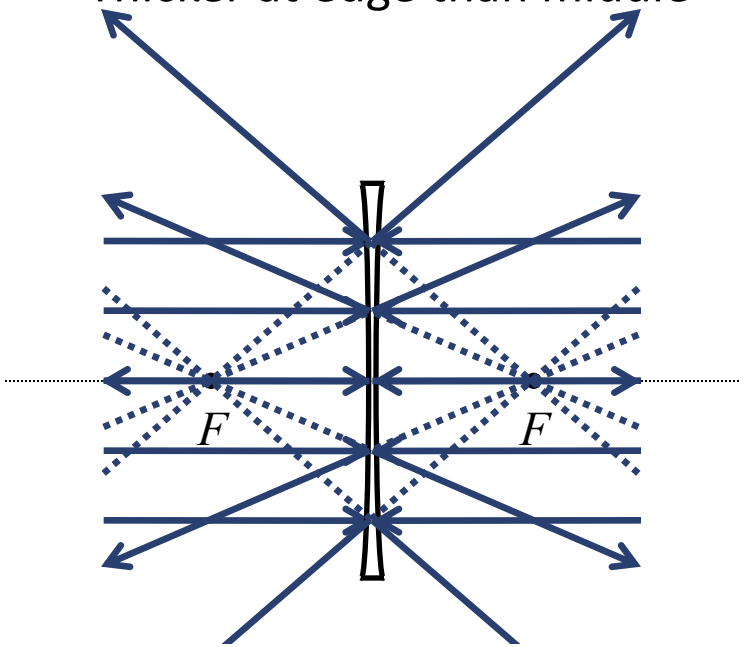
- Each incident ray will be refracted (using Snell's Law to find the angle of refraction)

Thin Lenses

Unlike mirrors, a thin lens has **2 focal points (one on each side)**

- Diverging (concave) Lens

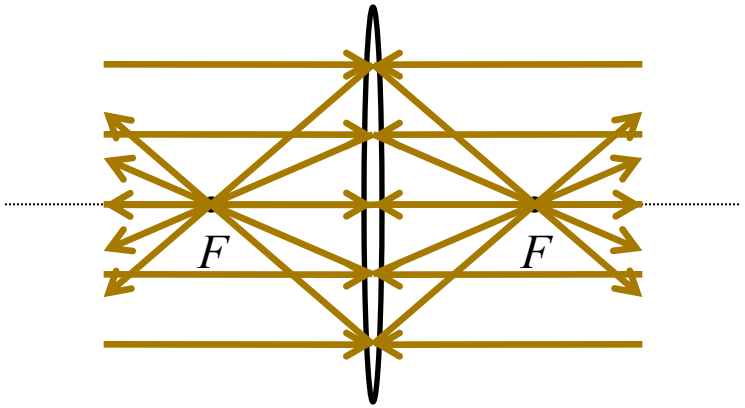
Thicker at edge than middle



Parallel rays appear to **diverge** from focal point

- Converging (convex) Lens

Thicker at middle than edge

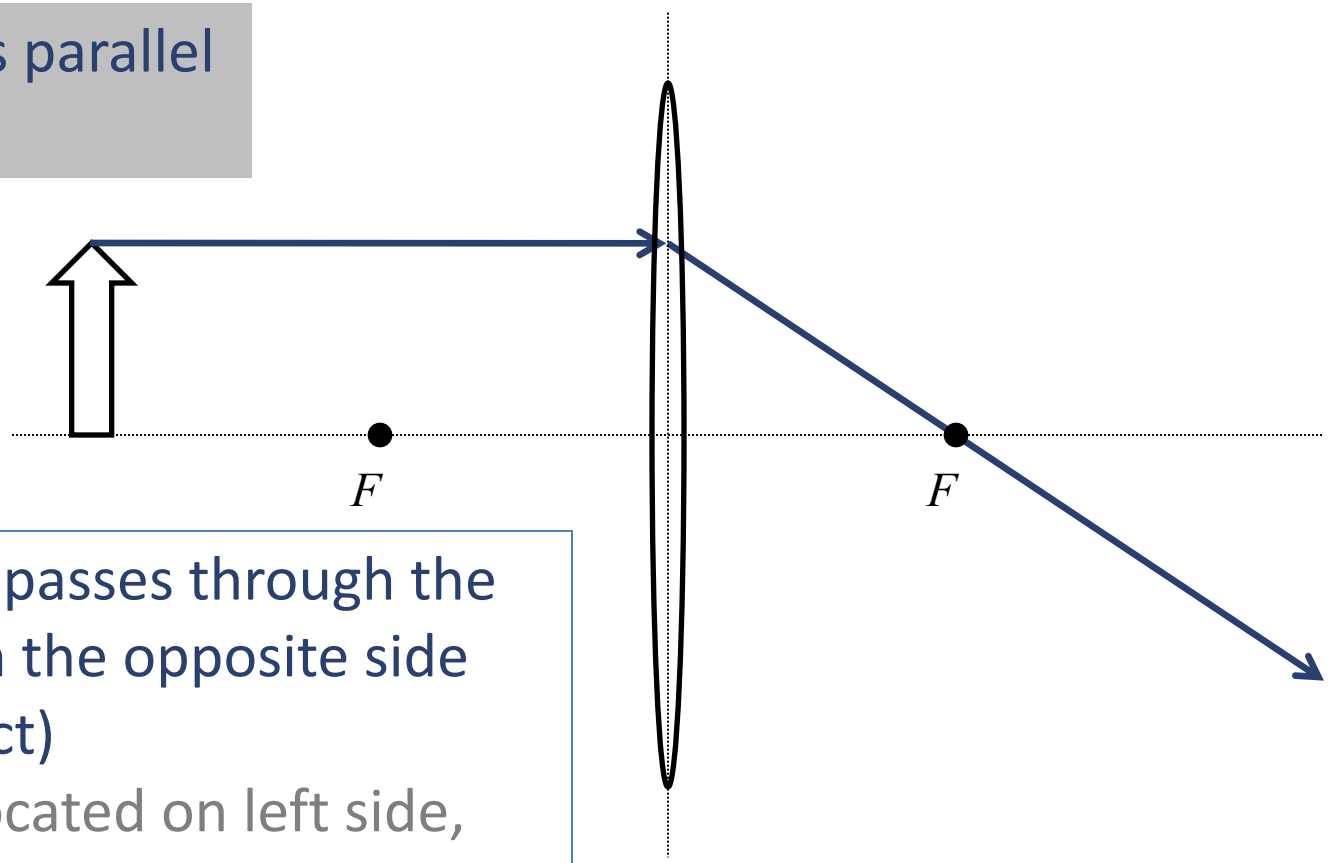


Parallel rays **converge** on focal point

Converging Lens

Special Ray 1

1. Ray which is parallel with the OA



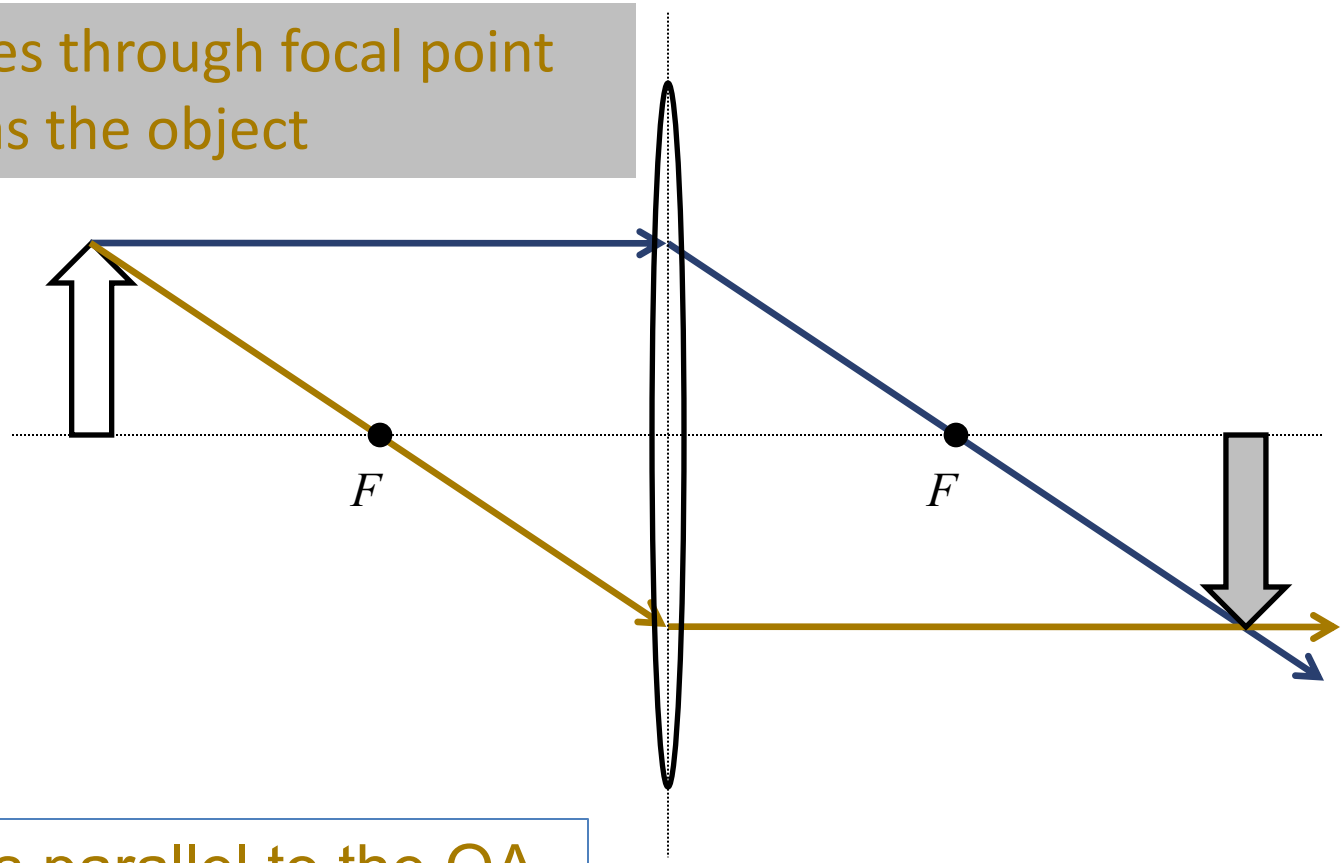
Refracted Ray passes through the focal point (on the opposite side from the object)

i.e. object is located on left side, and ray passes through F on right side

Converging Lens

Special Ray 2

2. Ray which passes through focal point on the same side as the object

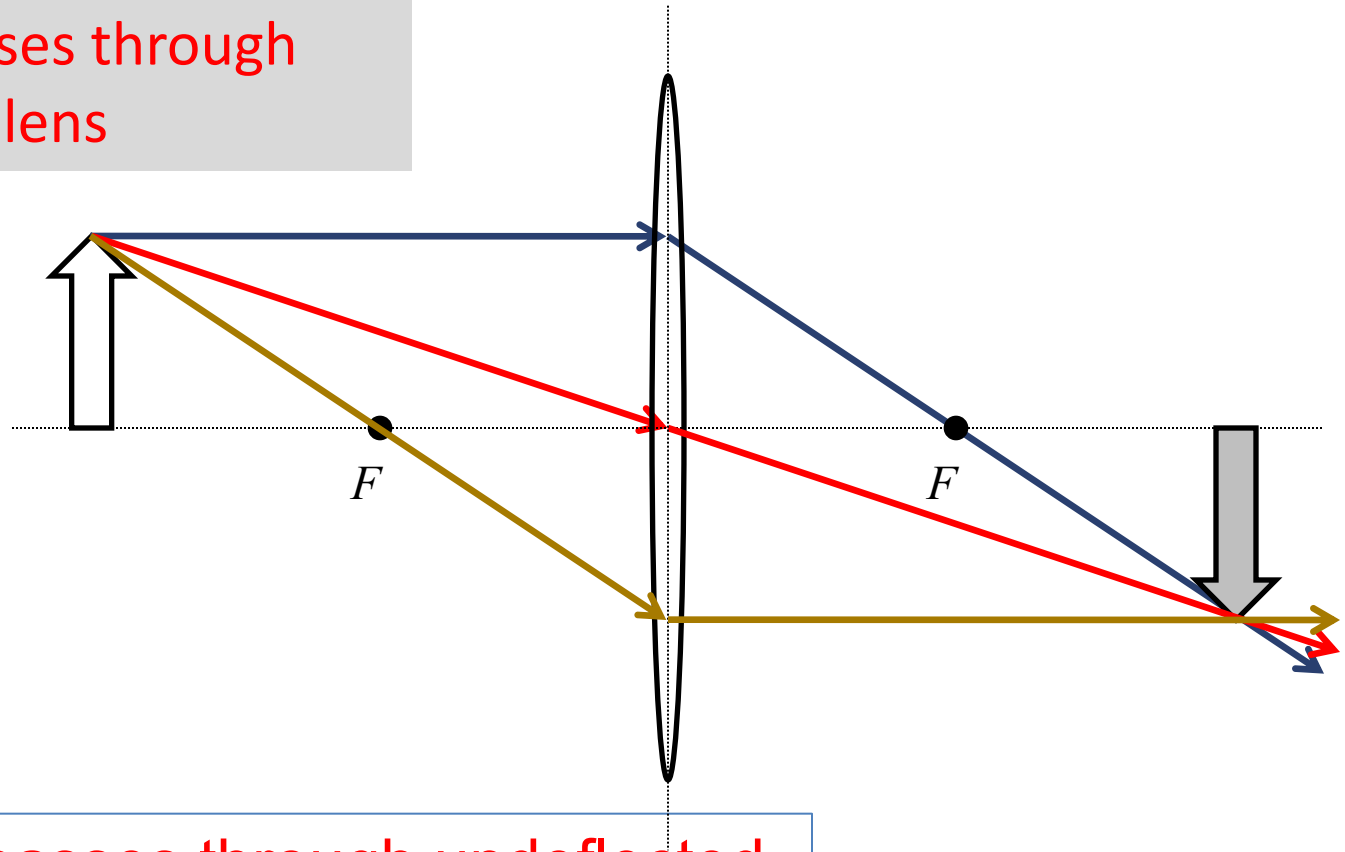


Refracted Ray is parallel to the OA

Converging Lens

Special Ray 3

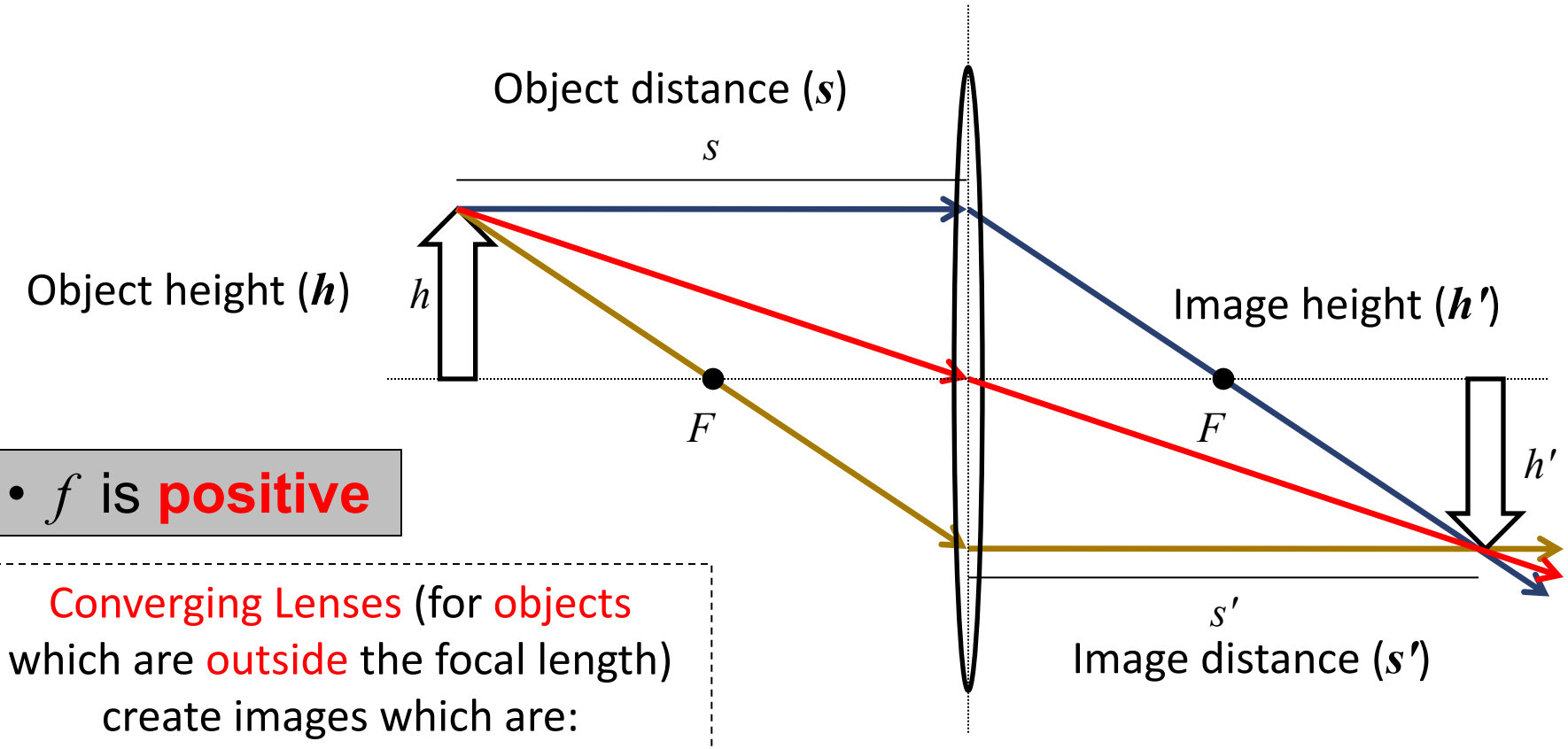
3. Ray which passes through the center of the lens



Refracted Ray passes through undeflected (because it is a **thin** lens)

Converging Lens – Summary

*(for object located **outside F**)*

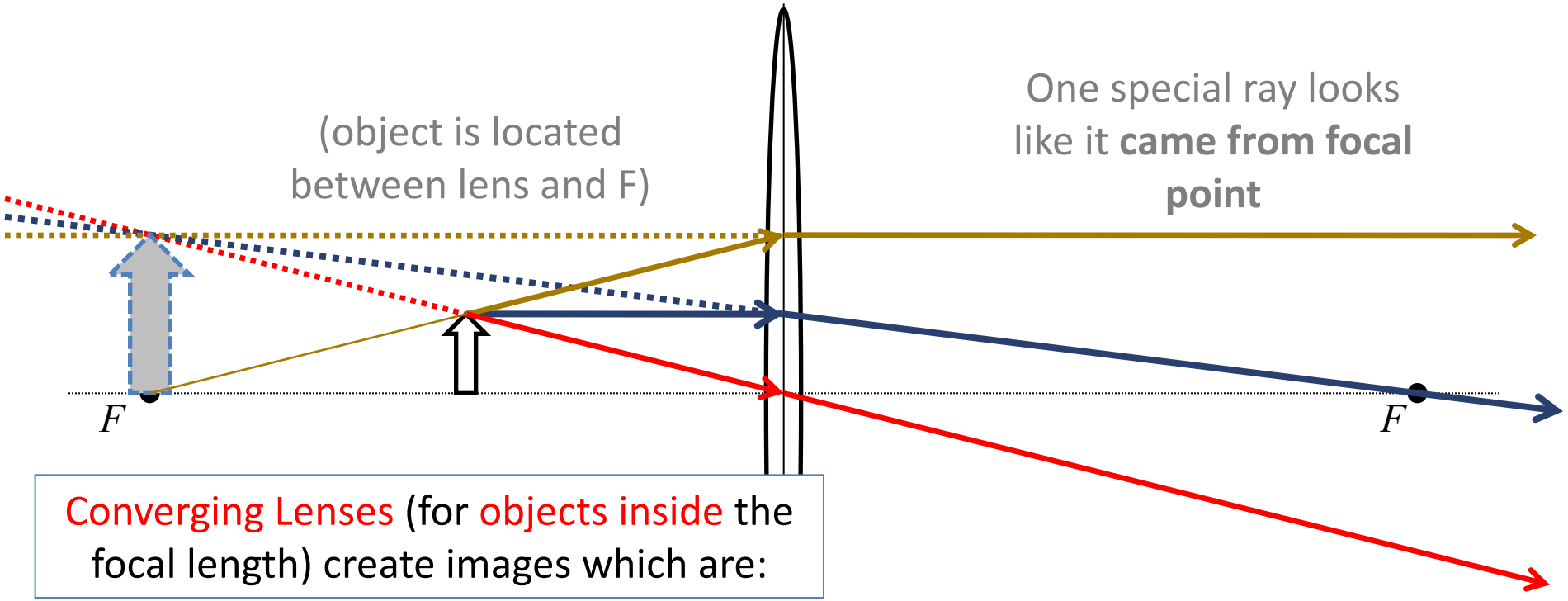


- f is **positive**

Converging Lenses (for objects which are outside the focal length) create images which are:

- **Inverted** (upside down)
- **Real** (rays converge at image)
- **Can be reduced, enlarged or same size** (depending on object location)

Converging Lens
*(object located **inside F**)*



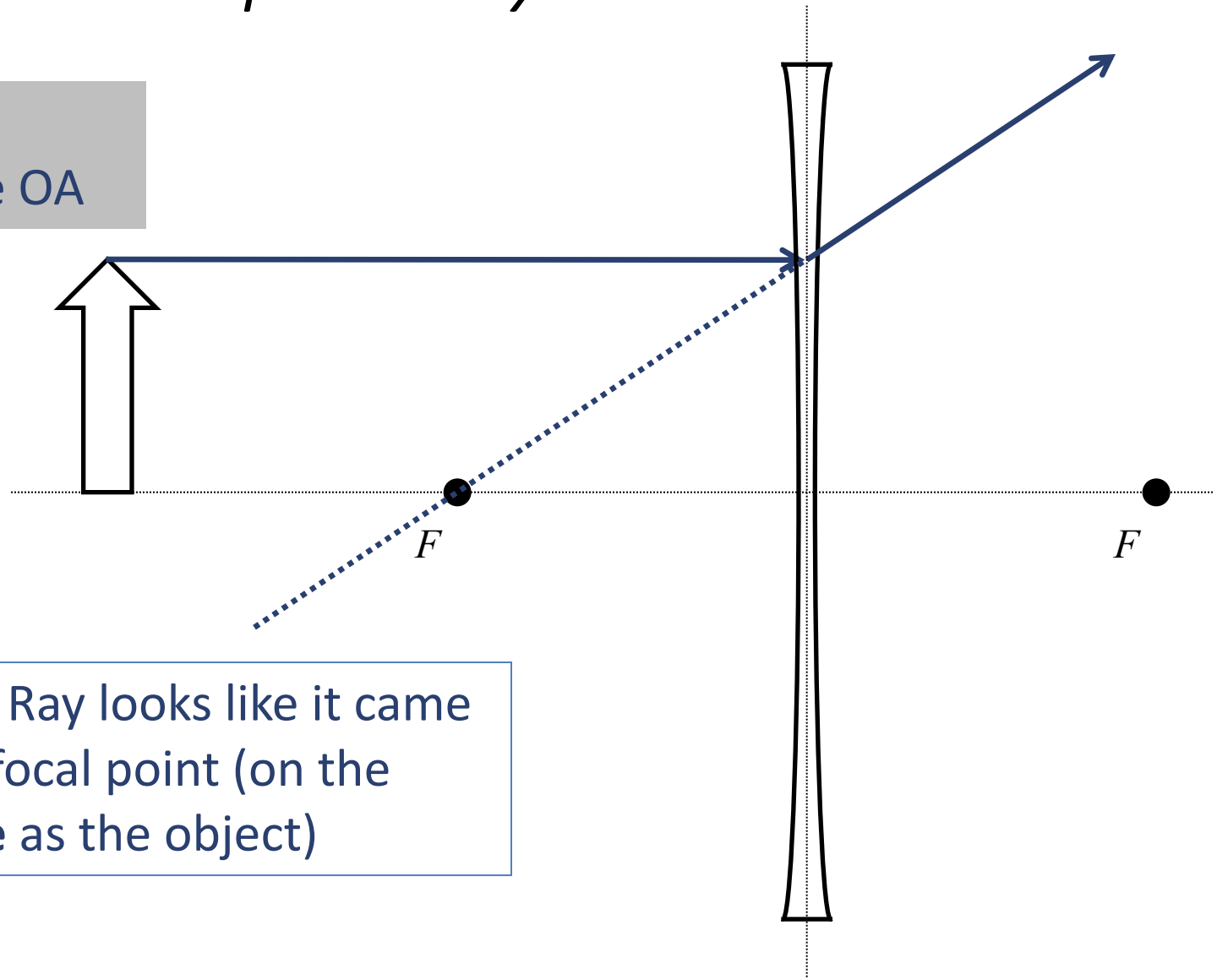
Converging Lenses (for objects inside the focal length) create images which are:

- **Upright** (no orientation change)
- **Enlarged**
- **Virtual** (actual rays don't converge)

Diverging Lens

Special Ray 1

1. Ray which is parallel with the OA

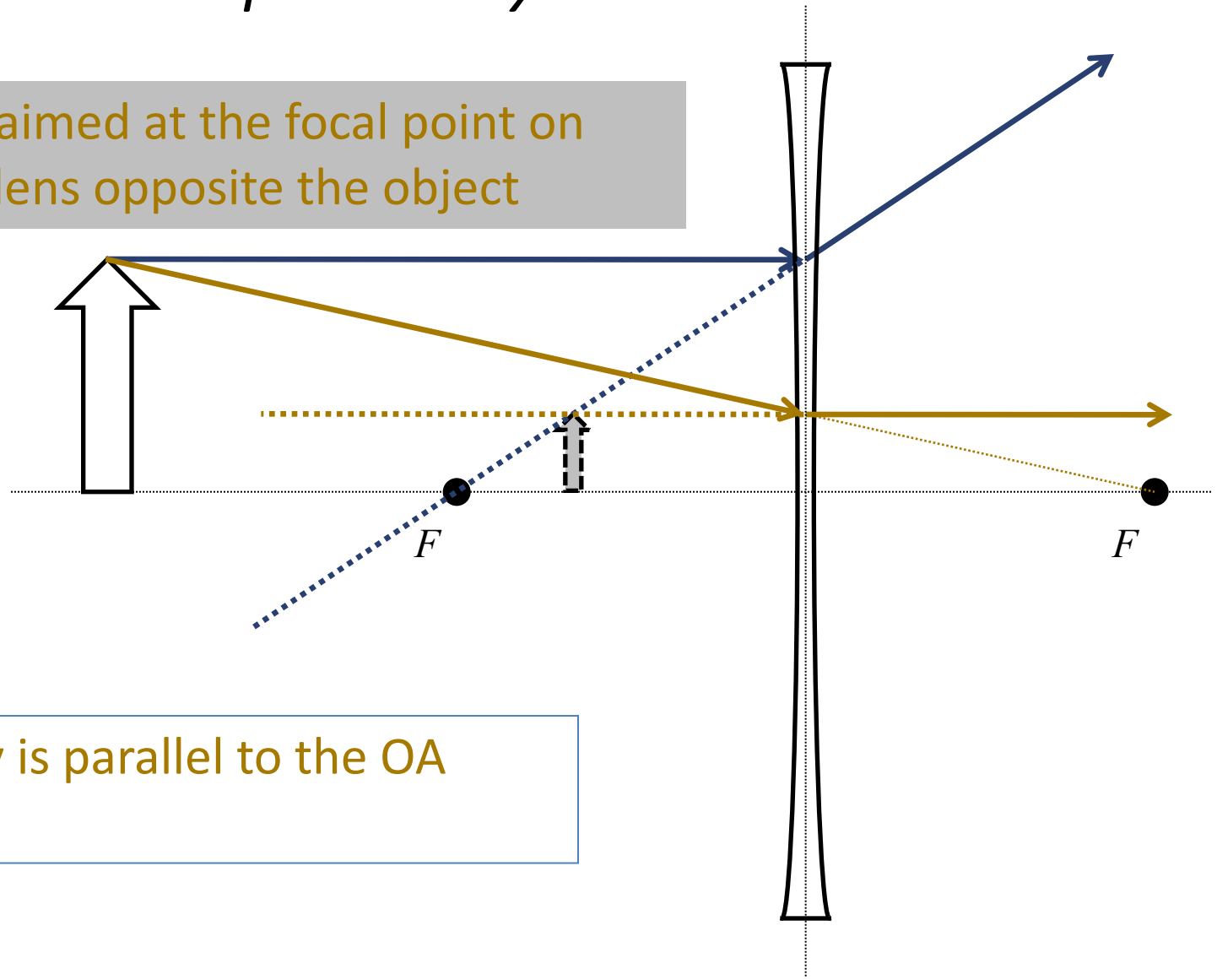


Refracted Ray looks like it came from the focal point (on the **same side** as the object)

Diverging Lens

Special Ray 2

2. Ray which is aimed at the focal point on the side of the lens opposite the object

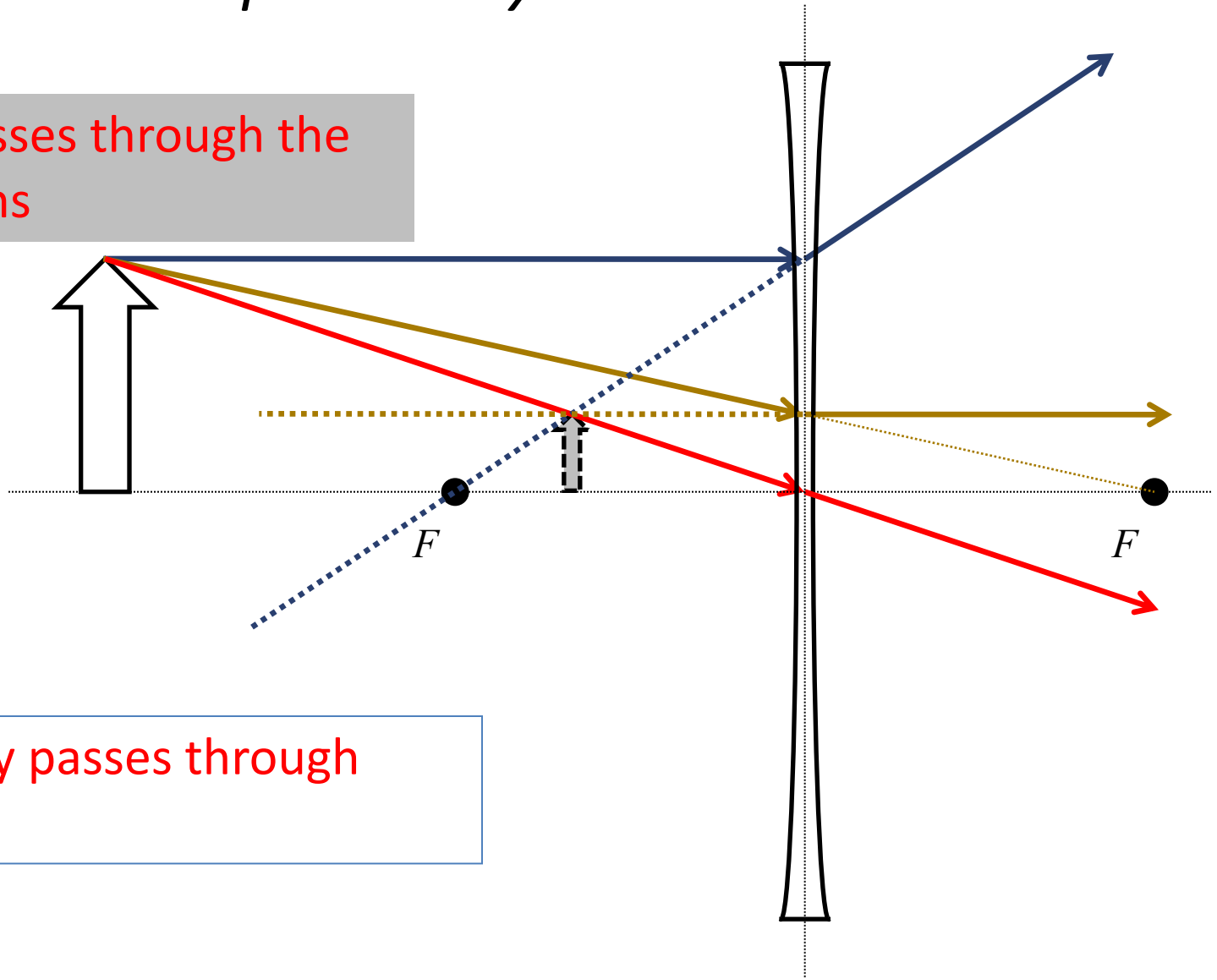


Refracted Ray is parallel to the OA (optical axis)

Diverging Lens

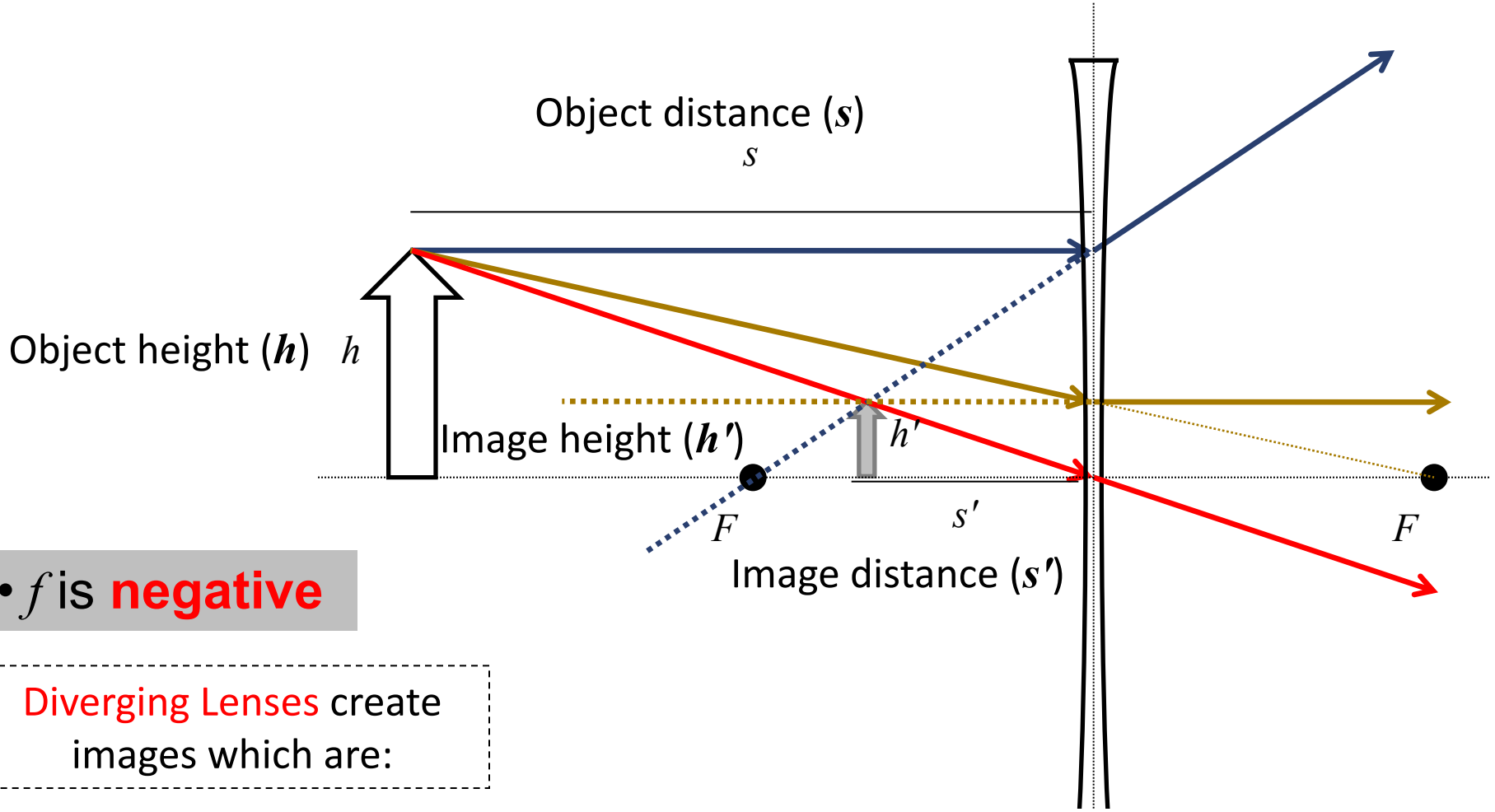
Special Ray 3

3. Ray which passes through the center of the lens



Refracted Ray passes through undeflected

Diverging Lens - Summary



- f is **negative**

Diverging Lenses create images which are:

- **Upright** (no orientation change)
- **Reduced** (smaller than object)
- **Virtual** (rays don't converge at image)

Ray Diagram Reminders

Draw rays all the way to **central line** of lens or **mirror plane**

(Since our ray diagrams only work for **thin** lenses and fairly flat mirrors)

Use at least two of the special rays:

1 – Starts parallel to OA

2 – Aimed at or from appropriate focal point (near focal point for converging lens; far focal point for diverging lens)

(These two rays are good enough for most sketches)

3 – Aimed at center of lens or mirror

Real vs. Virtual Images

- **Real images** occur when light rays converge at a point
- **Inverted** (if only one lens or mirror is present)
- **Can be used to project** an image on a screen
- Real images form on **far side** of lens from object or **in front** of mirror

---

- **Virtual images** occur when light rays appear to diverge from a point (but the projected rays converge to a point)
- **Upright** (if only one lens or mirror is present)
- **Cannot be used to project** an image onto a screen
- Virtual images form on **same side** of lens as object or **behind** mirror

Thin-Lens Equation

- OpenStax Chapter 25.6

Sign Conventions for Lenses

	Positive	**Negative**
Object Distance (s)	Object is **in front** of lens (usually)	
Image Distance (s')	Image is **real** (rays **pass though**)	Image is **virtual** (rays **don't pass through**)
Focal Length (f)	**Converging Lens**	**Diverging Lens**

Positive: someplace light is **actually converging or coming from**

Negative: someplace light **looks like it diverges from**

Thin Lens Equation

Ray diagrams give a relatively quick and powerful way to find information about an image
(e.g. **real vs. virtual**; **upright vs. inverted**; **size**)

Sometimes a mathematical argument will make life easier:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Magnification

As with mirrors (see lecture 1-2, slide 52):

$$m = \frac{h'}{h} = -\frac{s'}{s}$$

Magnification by Multiple Lenses

- Analyze multiple lenses by looking at each lens separately and calculating its magnification
- The total magnification from a series of lenses is the product of magnification of each lens:

$$m_{total} = m_{1st_lens} \times m_{2nd_lens} \times m_{3rd_lens} \cdots$$

For Exact Answers

Draw ray diagram to find if image is
real or virtual; upright or inverted; enlarged or reduced

Thin Lens Equation:
(Identical to Mirror Equation)

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Magnification:
(Same as for mirrors)

$$m=\frac{h'}{h}=-s'/s$$

Multiple lenses?

- Start with lens closest to object to find Image 1
- Use Image 1 as the object for the second lens
- Continue

Magnification is product of each lens's:

$$m_{total}=m_1\times m_2\times m_3\times\ldots=\frac{h'_{final}}{h_{object}}$$

Using the Thin Lens Equation

A **2.0 cm** tall object is near a lens. If its image is **upright** and 3.0 cm from the lens but only **1.5 cm** tall, what is the lens's focal length (including sign)?

~~a. +6.0 cm~~
b. –1.0 cm
~~c. +1.7 cm~~
~~d. +5.0 cm~~
e. –12 cm

Given: h = 2.0 cm

h' = +1.5 cm

Magnification: $m = h'/h = 0.75$

s' = –3.0 cm

Since image is reduced ($|m| < 1$) and upright ($m > 0$), it must be a **diverging** lens.

Therefore, f < 0 and s' < 0 (virtual image)

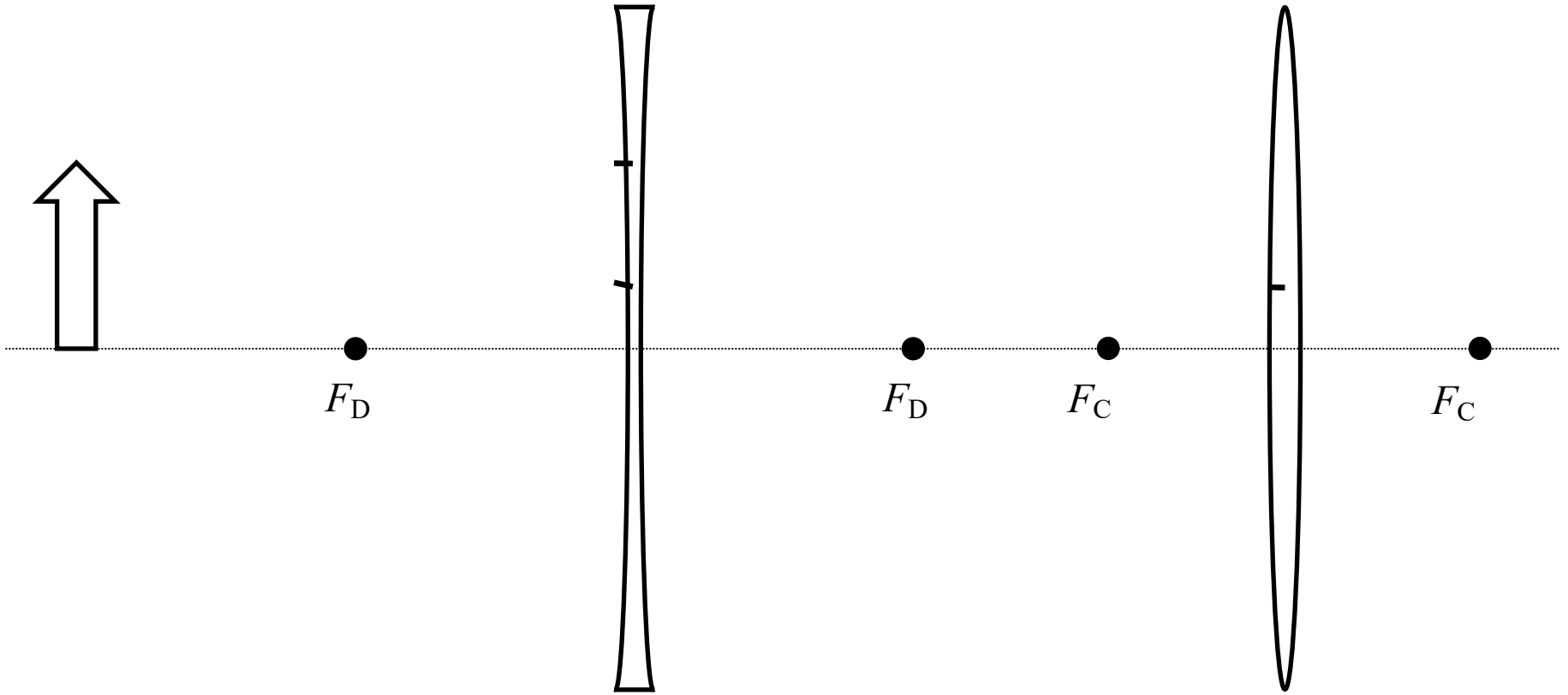
$$m = -\frac{s'}{s} \Rightarrow$$

s = +4.0 cm

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow$$

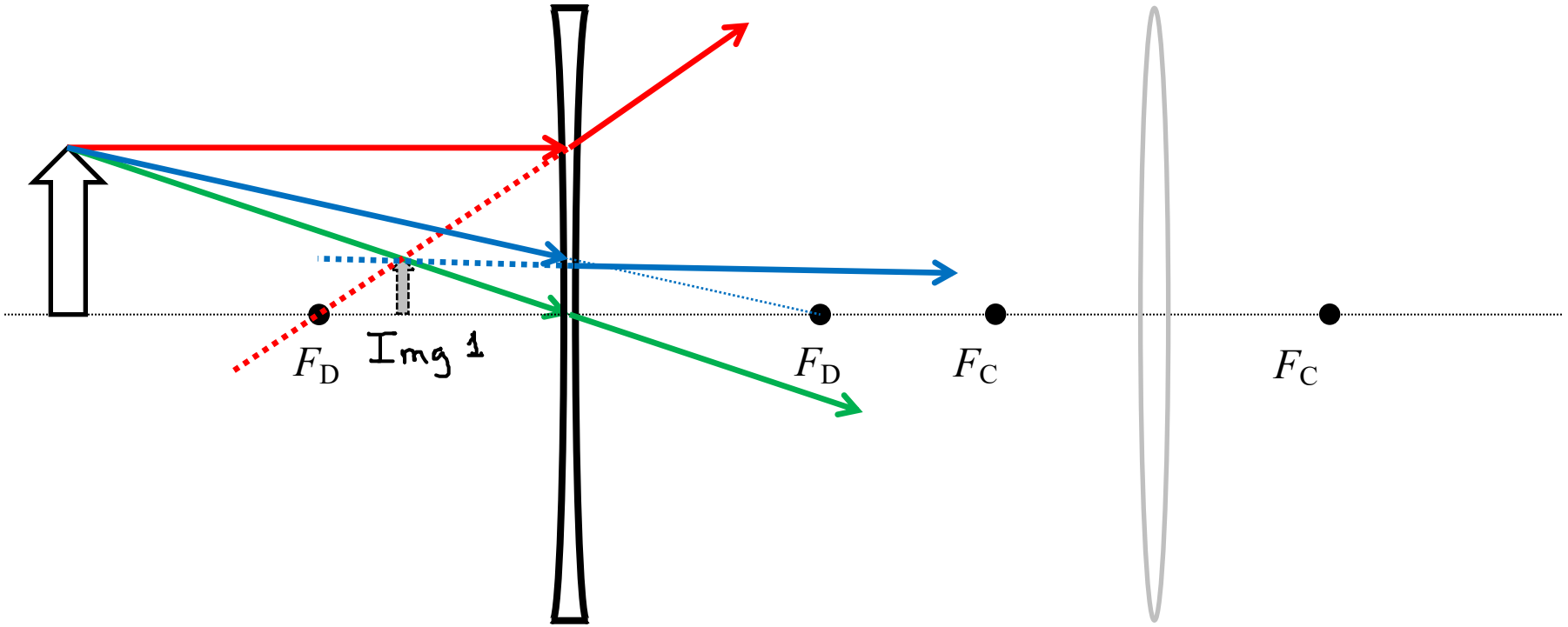
f = –12 cm

Ray Diagram with Two Lenses



Ray Diagram with Two Lenses

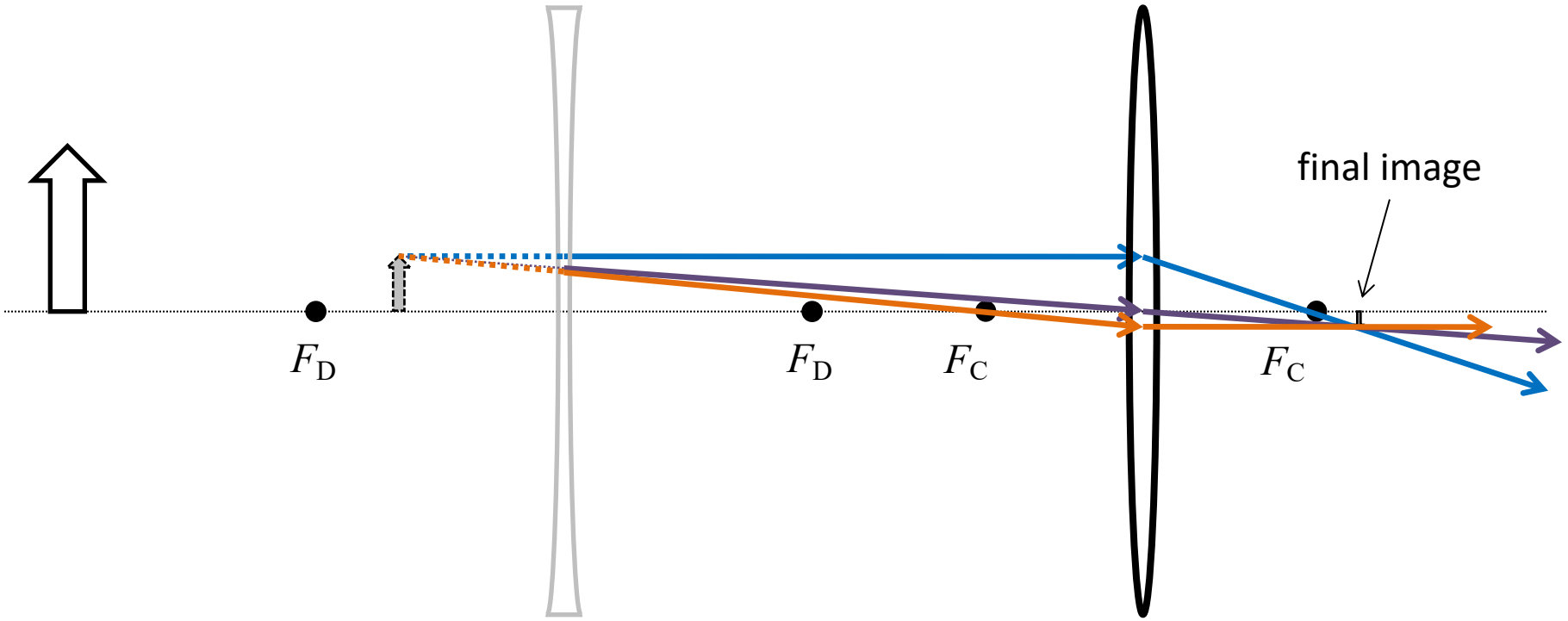
- If you have two lenses, you can still make a ray diagram.
- Find the image formed by one lens at a time.



- Use the **diverging lens** and its three special rays to find the image formed by just **that lens**

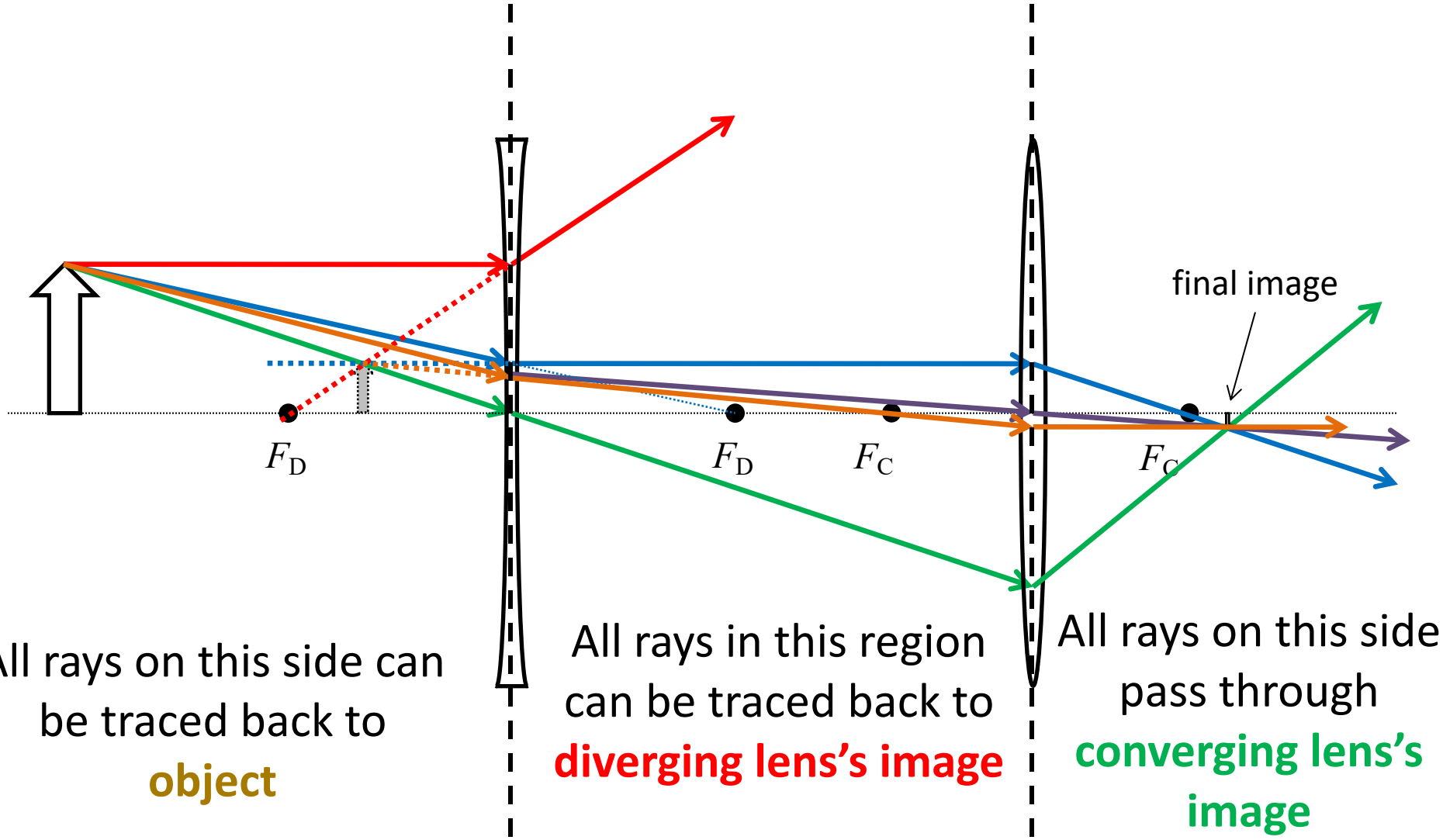
Ray Diagram with Two Lenses

- Draw special rays from the **diverging lens' image** to the **converging lens** and find the **final image**



Ray Diagram with Two Lenses

Complete Ray Diagram



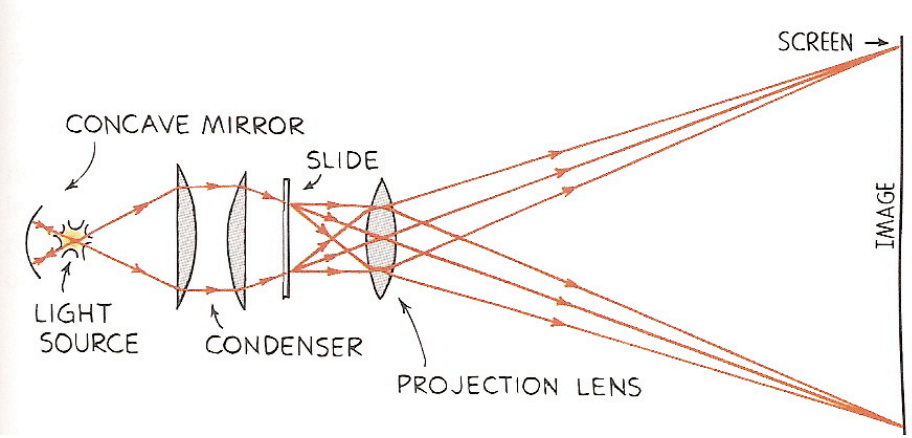
All rays on this side can be traced back to **object**

All rays in this region can be traced back to **diverging lens's image**

All rays on this side pass through **converging lens's image**

Application of Lenses

- Magnifying Glass (converging lens)
- Projector (converging lens and concave mirrors)



- Camera, eye